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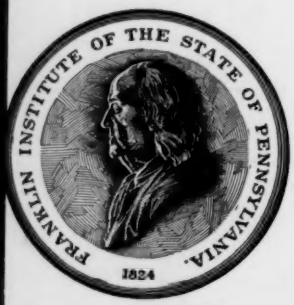
1826



1957

Deliberate Speed
HUGH TAYLOR

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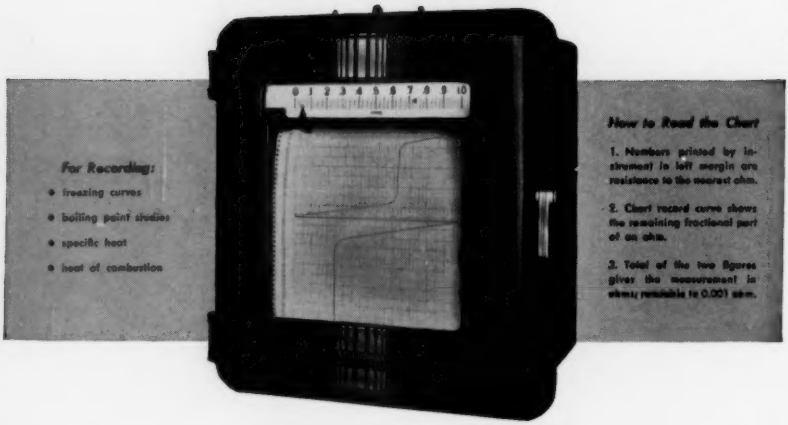
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• specific heat
• heat of combustion
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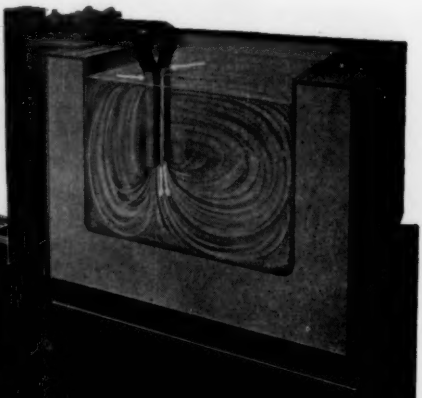

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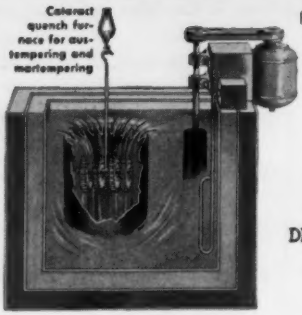

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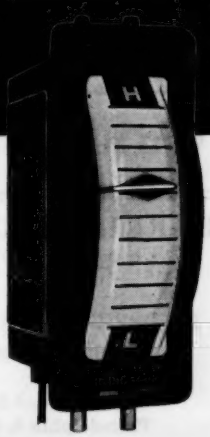

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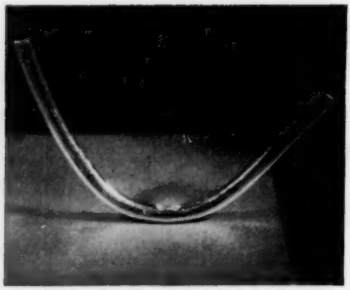
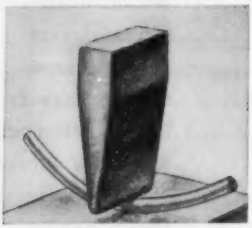


Wire bonded to germanium by thermo-compression technique (enlarged). Wires only 1/10 the breadth of a human hair have been successfully anchored to germanium wafers only three hairs thick. The bond may be an ohmic contact or rectifying contact by adding suitable impurities to the wire and the semiconductor.



One method of thermo-compression bonding. A heated wedge presses a wire against a heated semiconductor with moderate force. Adhesion occurs in seconds.

new way to join metal to semiconductors

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At Bell Labs Howard Christensen and Orson Anderson discuss their discovery of new bonding principle with Peter Andreatch, Jr., who collaborated in the studies.

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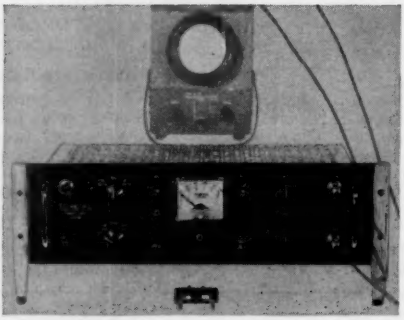

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The Stuart Ballantine Medal (1946—Gold Medal).—This medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

The Boyden Premium (1859).—This premium is awarded not oftener than once in five years to any resident of North America who has recently made a notable experimental determination of the speed, in free space, of radiation in any region of the entire spectrum.

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The Certificate of Merit (1882).—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

For further information relating to these awards apply to the Secretary, Committee on Science and the Arts

Journal
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Devoted to Science and the Mechanic Arts

Vol. 264 DECEMBER, 1957 No. 6

DELIBERATE SPEED *

BY

HUGH TAYLOR [1]

I fled him down the nights and down the days
I fled him down the arches of the years.

It is thus that Francis Thompson, the English poet of the last decades of the 19th century, described a flight from *The Hound of Heaven*; and of the pursuit he records that it took place "with unperturbed pace, deliberate speed, majestic instancy."

The phrase "with all deliberate speed" has recently been used by a Justice of the Supreme Court in a definition of the tempo with which a decision of the Court involving far-reaching social readjustments should be put into effect. It is a phrase which the catalytic chemist may appropriate also to define his activities in the field of chemistry. Of themselves, chemical species have their own rates of interaction. It is the objective, the privilege, of the catalytic chemist deliberately to alter such speeds to good ends, either to hurry their achievement for easier, larger yields or to slow them down, to accelerate or retard. The science of deliberate speed extends across one hundred and fifty years from Faraday's experimental researches on electricity through Langmuir to the nuclear age which we now are entering. In this age, the speeds range from the infinitesimal interval of the nuclear explosion to the deliberate speed of nuclear energy release in power plants for peaceful purposes.

Anyone acquainted with the railway journey, say on the California Zephyr, from Denver to Salt Lake City across the Colorado Rockies,

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(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

has an analogue of a catalytic chemical reaction. The train does not make a frontal attack on the intervening mountains. Rather, it wends its way gradually upwards along the valley floors higher and higher. Formerly, it was necessary for the train to ascend as far as the lowest of the mountain passes separating the eastern and western slopes of the mountains. With the construction of the Moffat Tunnel at a height of a few thousand feet above Denver it is now possible to pass through the tunnel in less than ten minutes and avoid the former tedious several-hour climb to the mountain pass. The Moffat Tunnel has catalyzed the Denver–Salt Lake run. It has provided a quicker, less energetic, alternative path from the one to the other city. It is equally available in either direction. Just so, a chemical catalyst provides an energetically easier, and therefore more rapid, pathway from reactants to products but equally well from products to reactants. The mountains between the two cities are the analogue of the potential energy barrier between the beginning and end products of the chemical process. And every chemical reaction, by a beneficent Boltzmann principle applicable in all chemical processes, finds the energetically easiest pathway. It is the job of the catalytic chemist to devise ever easier pathways, to explore, to discover the energetically lowest pass through the energy barrier that separates his raw material from his desired end products.

Nature is the catalytic chemist's most effective mentor. Nitrogen is one of the most inert of the chemical molecules. Within the last fifty years the catalytic chemist has learned how to unlock the molecule at iron surfaces and thus made possible the synthesis of ammonia from the nitrogen of the air throughout the civilized world. He achieves the loosening process at about 450 degrees Centigrade. And he needs high pressures of hydrogen-nitrogen mixtures to make a practicable yield of ammonia. In contrast, the nitrifying bacteria and leguminous nodules in the ground have ways to open up the nitrogen molecule at ground temperatures and normal pressures. Such examples serve to keep the catalytic chemist humble while inviting his successors to further effort. What bacteria can do at room temperature surely man will some day be capable of imitating!

Everyone taking a beginning course in chemistry is aware that two volumes of hydrogen and one volume of oxygen combine to form water, H_2O. Nowadays a freshman chemistry student may be asked to determine how rapidly a mixture of the two gases will react. If he stores a mixture over mercury in a measuring tube he will observe that the rate of combination is very slow. If the teacher asks him to calculate how soon he could observe a change in volume due to reaction of the mixture, say by one thimbleful, given that a million molecules of oxygen were disappearing to form water per second, the bright student could eventually come up with the startling answer that it

would take about one million years to observe such a change in volume. That speed is too deliberate for most people.

When Faraday produced a mixture of hydrogen and oxygen by the electrolysis of water he observed that the mixed gases when left in contact with the clean platinum electrodes did decrease steadily in volume. He came thereby to his remarkable researches on the power of clean metal surfaces to occlude—we would say "adsorb" or "chemisorb" gases and cause them to react. That was the first penetrating approach to the theory of catalytic action.

It was Langmuir in the second decade of the present century, working with clean tungsten filaments, who showed that the forces observed by Faraday were chemical forces and that the laws of chemical reaction could be formulated in terms of the residence of reacting gases on the catalyst surfaces.

Many other substances than these metals have the power to chemisorb hydrogen and oxygen. Princeton researches showed that oxides like zinc, chromium and manganese oxides would bind hydrogen chemically, at room temperatures or higher, while alumina and silica require temperatures as high as 500 degrees Centigrade before they will chemisorb hydrogen.

Alyea and Haber showed that crossed streams of heated hydrogen and oxygen would not ignite of themselves but a silica rod inserted in the crossing point of the gases caused inflammation. It was after this interesting experiment that Alyea demonstrated at Princeton the capacity of silica to take up large amounts of hydrogen when heated to the combustion temperature of hydrogen and oxygen.

The combustion which starts at the silica rod is propagated into the whole mixture when it inflames. That is because the reaction of hydrogen and oxygen to form water is a *chain reaction*. It only requires a trigger to set it off. Once started, under favorable conditions it is self propelling. Surfaces such as platinum, tungsten or silica, a lighted match, ultraviolet light, alpha particles, protons, neutrons all can be employed as triggers for the process. Indeed the hydrogen-oxygen reaction became the prototype of the branched chain reaction, one in which active intermediates like hydrogen atoms and oxygen atoms not only react but multiply themselves in the process, so that one produces two, two four, four eight, 8, 16, 32, 64, 128, 256, 512, 1024, in ten generations, more than one million in 20 generations, more than a billion in 30 generations.

And when this multiplication process occurs within a millionth of a second or less and when the energies involved change from the moderate energies of a conventional chemical reaction to the energies of nuclear processes then the world crosses the frontier to the perils of, but also to the opportunities—the golden opportunities if we will have it so—of the nuclear age.

In any chain process of the type we have been considering anything which removes active intermediates from the chain of processes will cause a slowing down of the reaction. It is thus that ethyl lead acts in the burning of gasoline in the automobile. It is thus that preservatives act in suppressing the oxidation and other processes in photographic developing solutions, fats, oils, rubber, plastics which result in deterioration of the materials. It is the mechanism whereby the control-rods in a nuclear pile can slow down the nuclear reactions to deliberate speed because they remove neutrons in controlled amounts from the fission process, prevent the uncontrolled multiplication of the neutrons and thus make possible the harnessing of nuclear energy.

There are two ways in which a reaction can be slowed down. That which we have just considered is retardation by the use of inhibitors. These act only in chain reactions. In non-chain processes the only way to slow down a reaction is to remove from the system a powerful catalyst which, when present, causes a fast reaction and when absent allows the process to occur at its normal slow pace.

Catalysis has been the major concern of my scientific research for more than forty-five years and in many areas of the subject. It began in the area of classical physical chemistry where the ionic theory of solutions used the catalytic activity of acids as an assist in the definition of the degree of ionization. At the Nobel Institute in Stockholm, with Arrhenius, in 1912 the topic of neutral salt action was still an unexplained phenomenon. Arrhenius published my efforts with some reluctance in the *Proceedings* of the Nobel Institute since they pointed to a catalytic activity of undissociated acid molecules, greater than that of hydrogen ions in the case of the "strong" acids. Arrhenius's reluctance was justified when Henry E. Armstrong in England, like Kahlenberg in this country one of the die-hard opposition to the ionic theory, hailed my researches as treason within the very fortress of the ionists. Subsequent researches by others have provided a more acceptable framework of interpretation.

A year later, in 1913, I was introduced to the topic of photocatalysis and the catalytic action of alpha-particles in the Technische Hochschule in Hannover. Bodenstein had just defined the phenomenon of chain reactions in terms of his research with Dux on the classical photochemical reaction, the union of hydrogen and chlorine. At Bodenstein's request, my former professor, F. G. Donnan, prevailed upon me to transfer from Stockholm to Hannover and there we began the study of the first chain reaction induced by a swift-moving particle, the alpha-particles from radon and radium-C. We were the first to show the chain nature of the process, the production of many thousands of molecules of hydrogen chloride for every ion-pair introduced by the alpha-particles into the hydrogen-chlorine mixture. It is of interest to remember that the now-so-familiar concept of a chain process had

its origins in those two researches in Bodenstein's laboratory in the year before World War I.

My introduction to the subject of contact catalysis, the catalytic activity of solid surfaces, dates from the last phases of World War I. Britain was making strenuous efforts to learn how to synthesize ammonia lest the success of the submarine campaign might isolate her from Chilean saltpeter supplies. It was an heroic effort, seven British scientists struggling to learn how to make hydrogen catalytically (the water-gas conversion process), how to compress hydrogen-nitrogen mixtures, of a purity hitherto technically unknown, to several hundred atmospheres pressure and find a suitable catalyst to cause these gases to combine at temperatures of 400–500°C. One small section was exploring the oxidation of ammonia on platinum gauzes to yield oxides of nitrogen and nitric acid. Our efforts during the war years resulted in moderate pilot-plant trials but our data became the basis upon which Imperial Chemical Industries built their synthetic ammonia development at Billingham. My own assignment in this program was the production of cheap hydrogen for which the water-gas conversion process became the essential operation. First singlehanded and later with Rideal the essentials of the technical process and the appropriate catalysts were unfolded. It was during the last year of the war that Rideal and I wrote "Catalysis in Theory and Practice." The book was completed and the preface written on Armistice Day, 1918.

As someone has subsequently remarked, the book was "light" on theory. There was no theory and little experiment to confirm theoretical views beyond the concepts of Faraday and Langmuir, with an assist in mid-nineteenth century from Deacon, who had conceived a catalyst for the Deacon chlorine process by library research on energy states of metallic oxides and chlorides. Rideal and I determined to return to our universities and see what could be done to alter this imbalance of practice and theory. We planned to divide the fields of research between us. While we started that way, time revealed that we both wandered into and out of the territories of research assigned to the other, with advantage, I hope, to the progress ultimately achieved.

That was the beginning of the research program in the Department of Chemistry at Princeton, a program which from 1919 until a month ago has resulted in more than 80 Ph.D. theses and the collaboration of about half as many postdoctoral fellows from the five continents of the world. To all of these the honor paid to me today is in major part due. Without their help the progress which has been achieved would not have occurred.

The research program has entered all the varied areas that are included in the modern concept of catalysis. With a pioneer group that included Pease and Benton, Burns, Gauger and Beebe the basic measurements of adsorption of gases at catalytic surfaces as well as

the energetics of adsorption were begun. My sponsor of this evening, Dr. Harvey A. Neville, was one of the earlier workers concerned with the catalytic influence of alkalis on the reactivity of solid carbon with steam. His early researches are even today significant in the present approaches to the problem. By 1925 it was evident that the properties of solid catalysts could be more convincingly presented if one were to assume that the surface was heterogeneous, with a spectrum of active centers, the fraction which was active being a function of the reaction accelerated. This concept became the focus of catalytic research during the next five years.

By 1930 the researches of Benton at Virginia had revealed the influence of temperature on the nature of the adsorption of hydrogen on nickel. This led to our formulation of the concept of chemisorption involving an energy of activation, a revolutionary concept at the time, but subsequently assimilated into the total theoretical picture. The experimental attack on this problem revealed the widespread phenomenon of adsorption of hydrogen by zinc, chromium, manganese oxides, alumina and silica at successively higher temperatures, of significance in catalysis of methanol and higher alcohol synthesis, and in the initiation and suppression of chain reactions at the surfaces of containing vessels in oxidation processes.

The discovery of para-hydrogen by Bonhoeffer in 1929 and of heavy hydrogen by Urey and Urey's subsequent separation of the heavy nitrogen isotope offered the catalytic chemist new tools with which to explore the phenomena of surface catalysis. A most fruitful decade of catalytic research in the new Frick Laboratory at Princeton resulted from these developments. It was possible to show that it was the activation of the nitrogen molecule that was the slow and difficult process in ammonia synthesis on iron and other catalysts. Heavy hydrogen provided a new tool for the investigation of hydrocarbons since it permitted an exploration of the separate activities of catalytic surface toward the carbon-hydrogen and carbon-carbon bonds in hydrocarbons. The existence of dissociative chemisorption of saturated hydrocarbon molecules was decisively revealed. It is pertinent to observe that these theoretical studies of isotopic reactions at catalyst surfaces paved the way for a rapid resolution of two World War II problems. One was the conversion of long chain paraffin hydrocarbons catalytically into aromatic hydrocarbons such as benzene, toluene and xylene (a process which had baffled the scientists during World War I and was efficiently achieved on a large technical scale early in World War II). The other was the stripping of deuterium-containing gases from electrolytic hydrogen by interaction with steam at surfaces of platinum, nickel and other metals.

It was an industrial contact with the problem of deterioration in artificial leather fabrics which led to an extended study of the problem

of inhibition and retardation of chemical reactions. Early work by Anderson in Princeton showed that the stability of hydrogen peroxide in yellow glass containers with added inhibitors was due to the suppression of photo-reaction and of chain processes. It was Bäckstrom in Princeton who revealed the full import of chain mechanisms in the oxidation of sodium sulphite solutions and of acetaldehyde. He demonstrated chain lengths of tens of thousands even in reactions in dilute aqueous solutions. Alyea assisted him in the elucidation of the antagonistic action of catalytic accelerators and the added inhibitors. The latter acted by shortening the chains started by the former.

Photochemical studies were undertaken to obtain data on the activation of molecules paralleling that achieved at catalytic surfaces. It was in this way that the properties of atomic hydrogen produced in discharge tubes and by photosensitization with mercury were explored to compare with the activities of chemisorbed hydrogen. This work led to a considerable chapter on the reactivities of atoms and radicals. Just over 30 years ago, it was shown in these photochemical experiments that free radicals from the interaction of atomic hydrogen with ethylene or from the decomposition of such substances as lead tetraethyl or mercury dimethyl could initiate a chain reaction polymerization of ethylene. This is one of the occasions where the Princeton research program "missed the technical boat" since it was years later that an industrial organization in England showed that the free radicals could be produced by traces of oxygen and that, under suitable conditions of temperature and pressure, the polymerization process could be made to yield the useful plastic, poly-ethylene. It is a minor consolation that one of the Princeton Ph.D.'s, Dr. C. O. Strother, subsequently received honor for the part he played in the American technical development of the poly-ethylene process to meet the special needs in World War II.

It would be difficult to justify, in terms of technical advances directly secured, the research program which Princeton University has so lavishly sponsored through four decades of time and which has enlisted the research activity of more than 120 students and visiting fellows for one or more years of their scientific life. One can point, it is true, to various points of contact with technological advance that resulted from these researches. It was actually in the pages of the JOURNAL OF THE FRANKLIN INSTITUTE in 1922 that the high pressure synthesis of methanol and one of the catalysts that could be employed were indicated. This was several years before the technical achievement. The studies of inhibition with hydrogen peroxide, sodium sulphite and acetaldehyde oxidations were inspired by participation in the technical developments of preservation of artificial leather and the use of tetraethyl lead in gasoline. The discovery with my former student and present colleague, Turkevich, that the paraffin, heptane,

when passed over a chromium oxide gel catalyst yielded toluene was a happy accident arising from a challenge, by my good friend, P. C. Keith, to turn aside from my normal path of catalytic investigation and pass a hydrocarbon over a catalyst. Why heptane and chromium oxide were chosen to meet the challenge did not have its origin wholly in science. But the scientific discovery that resulted was indeed a happy one. World War II revealed some of the consequences. Then, too, the researches involving catalysis among isotopic molecules led directly to a swift solution of a problem of production of heavy water in the first years of World War II. Professor H. C. Urey, in that period, gathered the distinct impression from the Princeton workers in catalysis that catalytic chemists developing catalysts were like magicians pulling rabbits out of hats. That was not true. Three decades of hard work in the theory of catalysis went into that wartime development.

It is in such manner, in the more swiftly achieved technological development rather than in the solution of a specific problem, that the technical justification of basic scientific research must be sought. The educational process involved is also, it must be remembered, a branching-chain, a snowball process. Out across the continent, in Europe, Asia and Africa, centers of catalytic research involving children and grandchildren of the initial Princeton effort can be traced. To the degree that this proliferation has yielded a more penetrating theoretical comprehension of the fuzzy concepts of 40 years ago, to the extent that theory has entered into "Catalysis in Theory and Practice," to that extent the practice has become swifter and surer. No one can survey the recent swift penetration of catalysis into the processing of petroleum, the production of petrochemicals, the synthetic rubber and plastics industry without sensing a correlation between the advance in our theoretical equipment and the speed of our industrial progress. We cannot as yet forecast the ultimate catalyst, but our catalytic theories do involve that a genial discovery can more swiftly be brought to high potential.

An interesting year in the history of the science and technology of deliberate speed is, this evening, drawing to a close. It was a year ago tomorrow, on October 17, 1956, that Queen Elizabeth II, at Calder Hall in England, switched electricity from that nuclear power station into the national grid of the Central Electricity Authority in Britain. That event symbolizes the taming of the speed of the nuclear fission process to the energy needs of man. The process utilizes the same techniques of control of nuclear chain reactions that had emerged in a forty-year study of conventional chemical chains. As I saw that station in August of this year, I was informed that its operation had given far less trouble than the early days of operation of conventional power stations or chemical plants. By 1965, it is expected that 15

per cent of the total generating capacity and 25 per cent of the total electricity consumption of the United Kingdom will be from 19 nuclear stations with an installed capacity of 5000–6000 megawatts. Nuclear energy for peaceful purposes is a reality.

Less than two weeks ago, the USSR launched the first earth satellite into space, 184 pounds of intricate scientific equipment, placed in its orbit by the speed of conventional chemical processes deliberately accelerated to achieve the necessary escape from the earth's atmosphere.

The retardation of nuclear fission processes and the enormous acceleration of our ordinary chemical reactions are measures of the breadth of control of speed now available to the scientist. They suggest to the thoughtful man another aspect of speed, that of attainment of scientific and technological excellence. Between two World Wars the United States was transformed from a primitive industrial civilization to the most highly developed technological organism that the world has ever known. A sober review of the past two decades yields the conclusion that the tempo of the American program can even be exceeded. As we meditate in such wise there comes the desire, the resolution, to put forth all the effort that we can in support of the ideals that we cherish.

The Franklin Institute, in offering each year its accolade to those whom it has judged of merit, acts as an incentive, a catalyst indeed, to such renewed effort. It is my privilege, as the Franklin Medalist of 1957, to act as spokesman for those whom you have honored this year, to thank you for your generous recognition of the little that we have been able to achieve and to express our appreciation of the—I almost said princely, but that is possibly not the right adjective in a democratic society—and so I leave it simply as our appreciation of the hospitality with which, each year, you grace this occasion.

Conservation of Parity Law Tested on Free Neutrons.—University of Chicago and Argonne National Laboratory scientists have announced experiments showing that a basic assumption of physics—the law of conservation of parity—does not hold true in the radioactive decay of the neutron. Tests with cobalt atoms, made earlier this year at the National Bureau of Standards, hinted this might be so. In these experiments, electrons were released by decaying neutrons locked in the heart of cobalt nuclei.

The new studies are the first to test the parity law on free neutrons. With previous experiments at the National Bureau of Standards, Columbia University, and the University of Chicago in January, these neutron experiments eliminate a misconception believed true and universal for 30 years. Thus the Argonne-Chicago experiment opens avenues of research that were previously considered impossible.

The neutron is a basic particle found in the nucleus of almost every atom. Like the earth, it spins on an axis, but in either direction. It has no electrical charge, but decays into two electrically-charged particles: a positive proton and a negative electron.

According to the disproved parity theory, these electrons and protons are always emitted equally toward either pole in any given number of decaying neutrons. To simplify the experiment, the Argonne-Chicago scientists measured only the distribution of electrons. They found that only about 62 per cent as many electrons came off toward one pole as the other.

The experiment was conceived by Valentine L. Telegdi, associate professor of physics at the University of Chicago, and carried out at Argonne by Merle T. Burgy, R. J. Epstein, Victor E. Krohn, Theodore B. Novey, Sol Raboy, and G. Roy Ringe, all of its physics division.

A scientific paper on the findings was published in the October *Physical Review* under the title, "A Measurement of Beta-Asymmetry in the Decay of Polarized Neutrons."

The "slow" neutrons for the experiment came from one of 50 openings in the side of Pile-5, an atomic reactor located at the Argonne laboratories near Lemont, Illinois, some 25 miles southwest of Chicago. Neutrons whose poles pointed the same way, and which spun in the same direction, were selected from the pile's beam by a magnetic mirror developed by Morton Hammermesh, associate director of Argonne's physics division. These identical neutrons were then directed into a vacuum chamber some six feet long and three feet square. On the top side of this long metal box was a device for detecting fast electrons.

Practically all of the air inside the chamber had been pumped out so that the neutrons would not accidentally collide with air molecules and release electrons unrelated to neutron decay. To get statistics on whether emitted electrons preferred either pole, the scientists needed large numbers of neutrons decaying in the chamber. This presented a major difficulty. While neutrons decay, on the average, after 18 minutes existence, even the "slow" 5,000-mile-per-hour neutrons used traveled through the vacuum chamber in only a hundred-thousandth of a second.

The problem was solved by shooting a steady stream of 70-million neutrons per second through the chamber. This so increased the number of neutrons that happened to decay while passing through that disintegrations were recorded only seconds apart.

TRAJECTORY PROGRAMMING FOR MAXIMUM RANGE

BY

GEORGE LEITMANN [1]

ABSTRACT

Necessary conditions are derived for the existence of the optimum direction of thrust and angle of attack of a rocket travelling in the atmosphere subject to forces of gravity, aerodynamic drag and lift. A mechanism is assumed capable of bringing about instantaneously the optimum angle of attack, that is, of providing the angle of attack-time function which yields maximum range between two specified altitudes. A method of solution is outlined and a singularity arising in the case of launch from zero velocity is discussed.

In a recent paper[2] Fried and Richardson derived the necessary conditions for the existence of an optimum thrust direction of a rocket travelling *in vacuo*. Some importance attaches to the problem of obtaining the optimum angle of attack of a rocket flying within the atmosphere, that is, of determining the angle of attack-time function which will permit a rocket of specified thrust to reach maximum range.

In this discussion it will be assumed that the rocket is subject to the following forces (lift is coplanar and perpendicular to drag):

$$\begin{aligned} \text{Lift} &= L(v, y, \alpha) \\ \text{Drag} &= D(v, y, \alpha) \\ \text{Gravity} &= m(t)g(y) \\ \text{Thrust} &= T(t, y) \end{aligned} \tag{1}$$

where

v = velocity
y = altitude (above sea level)
α = angle of attack
m = mass
t = time

and the quantities defined in (1) are specified functions of the parameters in parentheses.

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[1] Formerly Michelson Laboratory, U. S. Naval Ordnance Test Station, China Lake, Calif.; now, Dept. of Engineering, University of California, Berkeley, Calif.

[2] B. D. Fried and J. M. Richardson, "Optimum Rocket Trajectories," *J. Appl. Phys.*, Vol. 27, p. 955 (1956).

Figure 1 shows the inertial coordinate system OXY, origin at the center of the earth, with respect to which the motion will be described. Here

r = earth's radius

θ = angle of trajectory inclination to local horizontal.

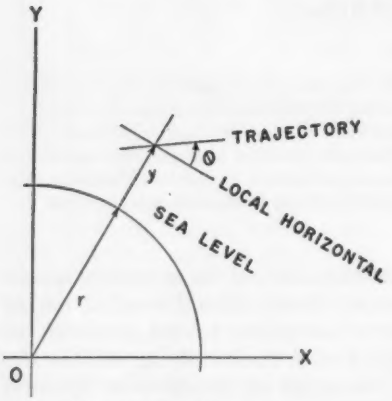

FIG. 1. Coordinate system.

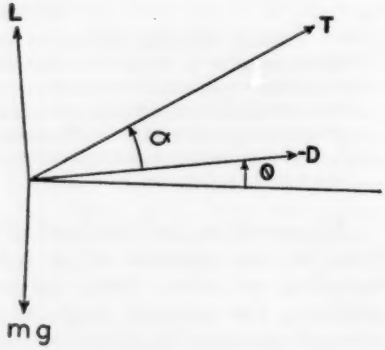

FIG. 2. Force system.

Figure 2 portrays the force system under consideration. If the thrust is taken along the axis of the rocket, the equations of motion of the rocket in the OXY plane are

$$\begin{aligned} m\dot{v} &= T\cos\alpha - D - mg\sin\theta \\ mv\dot{\theta} &= T\sin\alpha + L - mg\cos\theta + m\frac{v^2}{r+y}\cos\theta \\ \dot{y} &= v\sin\theta, \end{aligned} \tag{2}$$

where dot denotes differentiation with respect to time. The mechanism by which the optimum angle of attack is attained is not specified here. Both aerodynamic and thrust control is possible. At any rate, the equation of motion about the center of mass is not taken into account. In other words, it is assumed that whatever angle of attack is called for can be brought about instantaneously by the mechanism employed. Of course, this is an approximation which does not take into account the effect of the moment of inertia of the rocket and other time delays in the control system. If a control mechanism is specified, a parameter other than the angle of attack, say the deflection angle of the control vanes, can be optimized. In that case, the equation of motion about the center of mass must be utilized.

Throughout this paper, subscripts o and i will denote conditions at

the outset of flight and at impact, respectively. The range over the earth's surface (not including earth's rotation) is given by

$$r \int_0^{t_i} \frac{v \cos \theta}{r + y} dt. \tag{3}$$

The problem is then to find those functions $v(t)$, $y(t)$, $\theta(t)$, and $\alpha(t)$ for which the integral (3) is maximum subject to conditions (2). A word regarding boundary conditions is now in order. The initial speed, v_o, is specified. If $v_o = 0$, the initial direction of motion, θ_o, cannot also be given, for the initial angle of attack, α_o, is then a function of θ_o, and α_o may not be specified since $\dot{\alpha}$ does not occur in Eqs. 2 and 3. This can be seen from Eq. 2 with $v = 0$, that is,

$$\cos \theta = \frac{T}{mg} \sin \alpha.$$

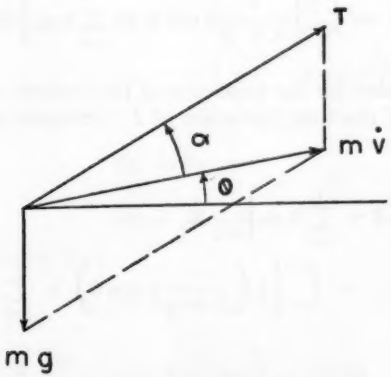

FIG. 3. Force system for $v = 0$.

It is also easily seen from Fig. 3 that α_o is a function of θ_o, since the initial direction of motion must be along $\dot{v}$ if $v_o = 0$. If $v_o \neq 0$, θ_o may be specified.

It will be assumed that v, θ, and y are continuous over the path. Since the thrust may be discontinuous, $\dot{v}$ and $\dot{\theta}$ may be discontinuous. No assumption regarding the continuity of α will be made at this time.

In general, for long-range rockets, it will be desired to find the optimum trajectory between two specified altitudes. For other types of missiles it may also be desirable to specify impact speed or trajectory angle, or both. Here, we shall deal with the case of specified initial and impact altitudes. (Extension to the case of other end conditions

is obvious.) Thus, the given boundary conditions are

$$\begin{aligned} &v = v_o, \qquad y = y_o \qquad \text{at } t = 0 \\ &y = y_i \qquad\qquad\quad \text{at } t = t_i. \end{aligned} \tag{4}$$

Equation 2 may be written

$$\begin{aligned} \phi_1 &\equiv \dot{v} - F(v, \theta, y, \alpha, t) = 0 \\ \phi_2 &\equiv \dot{\theta} - G(v, \theta, y, \alpha, t) = 0 \\ \phi_3 &\equiv \dot{y} - v \sin\theta = 0. \end{aligned} \tag{5}$$

Since integral (3) is to be maximized subject to conditions (5), we introduce undetermined multipliers λ_j, $j = 1, 2, 3$, assumed continuous over the path, and form the function

$$I \equiv \int_0^{t_i} \left[\frac{r}{r+y} v \cos\theta + \sum_{j=1}^{3} \lambda_j \phi_j \right] dt. \tag{6}$$

A necessary condition for the existence of the desired stationary solution is the vanishing of the first variation of I.[3] Since t_i is unspecified, this leads to

$$\begin{aligned} \delta I = &\left[\frac{r}{r+y} v \cos\theta + \sum_{j=1}^{3} \lambda_j \phi_j \right]_{t=t_i} \delta t_i \\ &+ \int_0^{t_i} \left[\delta \left(\frac{r}{r+y} v \cos\theta \right) + \sum_{j=1}^{3} \lambda_j \delta\phi_j \right] dt = 0 \end{aligned} \tag{7}$$

where

$$\delta \left(\frac{r}{r+y} v \cos\theta \right) = \frac{r}{r+y} \left(\cos\theta\, \delta v - v \sin\theta\, \delta\theta - \frac{v \cos\theta}{r+y} \delta y \right) \tag{8}$$

and

$$\begin{aligned} \lambda_1 \delta\phi_1 &\equiv \frac{d}{dt}(\lambda_1 \delta v) - \dot{\lambda}_1 \delta v - \lambda_1 \left(\frac{\partial F}{\partial v} \delta v + \frac{\partial F}{\partial \theta} \delta\theta + \frac{\partial F}{\partial y} \delta y + \frac{\partial F}{\partial \alpha} \delta\alpha \right) \\ \lambda_2 \delta\phi_2 &\equiv \frac{d}{dt}(\lambda_2 \delta\theta) - \dot{\lambda}_2 \delta\theta - \lambda_2 \left(\frac{\partial G}{\partial v} \delta v + \frac{\partial G}{\partial \theta} \delta\theta + \frac{\partial G}{\partial y} \delta y + \frac{\partial G}{\partial \alpha} \delta\alpha \right) \\ \lambda_3 \delta\phi_3 &\equiv \frac{d}{dt}(\lambda_3 \delta y) - \dot{\lambda}_3 \delta y - \lambda_3 (\sin\theta\, \delta v + v \cos\theta\, \delta\theta). \end{aligned} \tag{9}$$

[3] A. R. FORSYTH, "Calculus of Variations," New York, Cambridge University Press, 1927.

The following conditions are now imposed on the λ_j

$$\begin{aligned}
&\dot{\lambda}_1 + \lambda_1 \frac{\partial F}{\partial v} + \lambda_2 \frac{\partial G}{\partial v} + \lambda_3 \sin\theta - \frac{r}{r+y}\cos\theta = 0 \\
&\dot{\lambda}_2 + \lambda_1 \frac{\partial F}{\partial \theta} + \lambda_2 \frac{\partial G}{\partial \theta} + \lambda_3 v\cos\theta + \frac{r}{r+y} v\sin\theta = 0 \qquad (10) \\
&\dot{\lambda}_3 + \lambda_1 \frac{\partial F}{\partial y} + \lambda_2 \frac{\partial G}{\partial y} + \frac{r}{(r+y)^2} v\cos\theta = 0.
\end{aligned}$$

Since $\delta\alpha$ is arbitrary, its coefficient must vanish. Thus,

$$\lambda_1 \frac{\partial F}{\partial \alpha} + \lambda_2 \frac{\partial G}{\partial \alpha} = 0 \qquad (11)$$

and so, since $\sum_{j=1}^{3} \lambda_j \phi_j = 0$,

$$\lambda_{1i}\delta v_i - \lambda_{1o}\delta v_o + \lambda_{2i}\delta\theta_i - \lambda_{2o}\delta\theta_o + \lambda_{3i}\delta y_i - \lambda_{3o}\delta y_o + \left(\frac{r}{r+y} v\cos\theta\right)_{t=t_i} \delta t_i = 0. \qquad (12)$$

Since v_o, y_o, and y_i are given

$$\delta v_o = \delta y_o = 0$$

and (13)

$$\delta y_i = -\dot{y}_i \delta t_i.$$

Equation 12 then leads to

$$\lambda_{1i} = \lambda_{2i} = \lambda_{2o} = 0 \qquad (14)$$

and

$$\left(\frac{r}{r+y} v\cos\theta - \lambda_3 \dot{y}\right)_{t=t_i} = 0. \qquad (15)$$

We may note that, if $v_o \neq 0$, and θ_o is specified, $\delta\theta_o = 0$ and the condition $\lambda_{2o} = 0$ is replaced by $\theta = \theta_o$. If the thrust is discontinuous at burn-out, Eq. 11 shows that α is discontinuous, too.

The Euler-Lagrange equations, Eqs. 10 and 11, together with the equations of motion, Eq. 5, and the boundary conditions, (4), (14), and (15), constitute a system of equations for the determination of $v(t)$, $\theta(t)$, $y(t)$, $\alpha(t)$, and $\lambda_j(t)$, $j = 1, 2, 3$.

Before outlining a possible method of solution, a first integral of the Euler-Lagrange equations will be found. The integrand of integral (6) is

$$\ell \equiv \frac{r}{r+y} v\cos\theta + \lambda_1 \dot{v} - \lambda_1 F + \lambda_2 \dot{\theta} - \lambda_2 G + \lambda_3 \dot{y} - \lambda_3 v\sin\theta. \qquad (16)$$

During free flight, ℓ is not an explicit function of time, that is,

$$\dot{\ell} = \frac{d}{dt}(\lambda_1 \dot{v} + \lambda_2 \dot{\theta} + \lambda_3 \dot{y}) + \dot{v}\left(\frac{r}{r+y}\cos\theta - \lambda_1 \frac{\partial F}{\partial v} - \lambda_2 \frac{\partial G}{\partial v} - \lambda_3 \sin\theta\right) - \dot{\theta}\left(\frac{r}{r+y} v \sin\theta + \lambda_1 \frac{\partial F}{\partial \theta} + \lambda_2 \frac{\partial G}{\partial \theta} + \lambda_3 v \cos\theta\right) - \dot{y}\left(\frac{r}{(r+y)^2} v \cos\theta + \lambda_1 \frac{\partial F}{\partial y} + \lambda_2 \frac{\partial G}{\partial y}\right) - \dot{\lambda}_1 F - \dot{\lambda}_2 G - \dot{\lambda}_3 v \sin\theta. \quad (17)$$

In view of Eqs. 5 and 10, Eq. 17 becomes

$$\frac{d}{dt}\left(\frac{r}{r+y} v \cos\theta - \lambda_1 \dot{v} - \lambda_2 \dot{\theta} - \lambda_3 \dot{y}\right) = 0 \quad (18)$$

that is,

$$\frac{r}{r+y} v \cos\theta - \lambda_1 \dot{v} - \lambda_2 \dot{\theta} - \lambda_3 \dot{y} = C, \quad (19)$$

but in view of Eqs. 14 and 15,

$$C = 0. \quad (20)$$

Calculation may proceed in the following manner:

1. Assume sets of values for λ_1, λ_3, and θ (or λ_2 if θ_o is given) at $t = 0$ and integrate Eqs. 5 and 10 with Eq. 11 over the interval of powered flight (remembering that the burning time is specified). Each set of initial values will yield values for v, θ, y, α, and the λ_j at burn-out.

2. Since v, θ, y and the λ_j are assumed continuous, their burn-out values are equal to their values at the outset of free flight. Equation 11 then yields α at the beginning of free flight. The variables v, θ, y, and α can now be plotted as functions of λ_1 and λ_2 at the outset of free flight.

3. By means of Eq. 19 express λ_3 in terms of λ_1 and λ_2.

4. Choose sets of initial free-flight values for λ_1 and λ_2, and with the corresponding values of v, θ, y, and α integrate the equations of motion and the Euler-Lagrange equations until $y = y_i$. The desired set of starting values is that for which $\lambda_1 = \lambda_2 = 0$ at impact.

APPENDIX A

In order to integrate the equations of motion and the Euler-Lagrange equations by numerical methods, it is necessary to know not only the initial values of v, θ, y, and the λ_i, but also those of their first derivatives. When $v_o \neq 0$, the initial values of the first derivatives can be obtained directly from the differential equations. However, when $v_o = 0$, the function G, and hence $\partial G/\partial v$, $\partial G/\partial \theta$, $\partial G/\partial y$, and $\partial G/\partial \alpha$ become indeterminate. In what follows, this case will be discussed; $t \to 0$ will then imply $v \to 0$. In view of the equations of motion, the Euler-Lagrange equations may be written in the form

$$\begin{aligned}
\dot{\lambda}_1 &= \lambda_1 \frac{\partial \overline{D}}{\partial v} + \frac{\lambda_2}{v}\left(\dot{\theta} - \frac{\partial \overline{L}}{\partial v} - \frac{2v}{r+y}\cos\theta\right) - \lambda_3 \sin\theta + \frac{r}{r+y}\cos\theta \\
\dot{\lambda}_2 &= \lambda_1 g\cos\theta - \frac{\lambda_2}{v}\left(g\sin\theta - \frac{v^2}{r+y}\sin\theta\right) - \lambda_3 v\cos\theta - \frac{r}{r+y}v\sin\theta \\
-\dot{\lambda}_3 &= \lambda_1\left(\frac{\cos\alpha}{m}\frac{\partial T}{\partial y} - \frac{\partial \overline{D}}{\partial y} - \sin\theta\frac{\partial g}{\partial y}\right) \\
&\quad + \frac{\lambda_2}{v}\left(\frac{\sin\alpha}{m}\frac{\partial T}{\partial y} + \frac{\partial \overline{L}}{\partial y} - \cos\theta\frac{\partial g}{\partial y} - \frac{v^2}{(r+y)^2}\cos\theta\right) + \frac{r}{(r+y)^2}v\cos\theta \\
&\quad \times \lambda_1\left(\frac{T}{m}\sin\alpha + \frac{\partial \overline{D}}{\partial \alpha}\right) - \frac{\lambda_2}{v}\left(\frac{T}{m}\cos\alpha + \frac{\partial \overline{L}}{\partial \alpha}\right) = 0.
\end{aligned} \tag{21}$$

Use will be made of the following aerodynamic information. As $t \to 0$

$$D \sim v^2, \qquad L \sim v^2$$

so that

$$\begin{aligned}
D = L &= 0 \\
\frac{\partial D}{\partial y} = \frac{\partial L}{\partial y} &= 0 \\
\frac{\partial D}{\partial v} = \frac{\partial L}{\partial v} &= 0
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
\frac{\partial D}{\partial \alpha} = \frac{\partial L}{\partial \alpha} &= 0 \\
\frac{d}{dt}\frac{\partial D}{\partial \alpha} = \frac{d}{dt}\frac{\partial L}{\partial \alpha} &= 0.
\end{aligned} \tag{23}$$

From Eqs. 21_4 and 23 it follows that as $t \to 0$

$$\frac{\lambda_2}{v} = \lambda_1 \tan\alpha. \tag{24}$$

Furthermore, as $t \to 0$

$$\dot{y} = 0. \tag{25}$$

Application of l'Hospital's rule to Eq. 5_2, and use of Eqs. 23 and 24, lead to

$$\dot{\theta} = \left[\sin\alpha\frac{\partial}{\partial t}\left(\frac{T}{m}\right) + \frac{T}{m}\dot{\alpha}\cos\alpha\right](v - g\sin\theta)^{-1} \tag{26}$$

as $t \to 0$. In order to obtain an expression for $\dot{\alpha}$, Eq. 21_4 is differentiated. As $t \to 0$, this yields together with Eq. 24

$$\lambda_1\dot{\alpha}\sec\alpha + \dot{\lambda}_1\sin\alpha - \cos\alpha\frac{d}{dt}\left(\frac{\lambda_2}{v}\right) = 0. \tag{27}$$

But

$$\frac{1}{v}\frac{d}{dt}\left(\frac{\lambda_2}{v}\right) = \frac{\dfrac{\dot{\lambda}_2}{v} - \dfrac{\lambda_2}{v}}{v}. \tag{28}$$

By l'Hospital's rule, as $t \to 0$

$$\frac{\lambda_2}{v} = \frac{\dot{\lambda}_2}{\dot{v}} \tag{29}$$

so that again by l'Hospital's rule, as $t \to 0$

$$2\frac{d}{dt}\left(\frac{\lambda_2}{v}\right) = \frac{d}{dt}\left(\frac{\dot{\lambda}_2}{\dot{v}}\right). \tag{30}$$

Using Eqs. 5_1, 21_2, and 30 yields

$$\frac{d}{dt}\left(\frac{\lambda_2}{v}\right) = \left[2 + \frac{g\sin\theta}{\dot{v}}\right]^{-1}\left\{\frac{1}{\dot{v}}\left[\dot{\lambda}_1 g\cos\theta + \lambda_1\left(\frac{T}{m}\dot{\alpha}\sin\alpha\tan\alpha - \sin\alpha\frac{\partial}{\partial t}\left(\frac{T}{m}\right) - g\dot{\theta}\sin\theta\right)\right] - \lambda_3\cos\theta - \frac{r}{r+y}\sin\theta\right\}. \tag{31}$$

Also, as $t \to 0$

$$\begin{aligned}
\dot{v} &= \frac{T}{m}\cos\alpha - g\sin\theta \\
\dot{\lambda}_1 &= \frac{\lambda_2}{v}\dot{\theta} - \lambda_3\sin\theta + \frac{r}{r+y}\cos\theta \\
\dot{\lambda}_2 &= \lambda_1 g\cos\theta - \frac{\lambda_2}{v}g\sin\theta \\
\dot{\lambda}_3 &= \lambda_1\left(\sin\theta\frac{\partial g}{\partial y} - \frac{\cos\alpha}{m}\frac{\partial T}{\partial y}\right) + \frac{\lambda_2}{v}\left(\cos\theta\frac{\partial g}{\partial y} - \frac{\sin\alpha}{m}\frac{\partial T}{\partial y}\right).
\end{aligned} \tag{32}$$

As stated previously, when $t \to 0$

$$\frac{T}{m}\sin\alpha = g\cos\theta. \tag{33}$$

Thus, for every set of initial values of λ_1, λ_3, and θ, together with the initial value of y, Eqs. 24–27, 31, and 32, constitute a set of equations which can be solved for the initial values of the first derivatives.

APPENDIX B

It is both instructive and may serve as a check on the method employed to apply the results of the main section to the case of flight *in vacuo* over a flat earth. For this case only thrust direction is of interest, since angle of attack does not occur in the equations of free flight, and indeed has no meaning. This simple case may be treated in a straightforward manner [2,4] without the use of Lagrangian multipliers.

We wish to consider

$$\begin{aligned}
L &= D = 0 \\
r &= \infty \\
g &= \text{constant} \\
T &= T(t) \\
m &= m(t).
\end{aligned}$$

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[4] G. LEITMANN, "Optimum Programming of Thrust Direction," *Jour. Brit. Interplan. Soc.*, to appear.

The equations of motion are then

$$\begin{aligned} \dot{v} &= \frac{T}{m}\cos\alpha - g\sin\theta \\ v\dot{\theta} &= \frac{T}{m}\sin\alpha - g\cos\theta \\ \dot{y} &= v\sin\theta \end{aligned} \tag{33}$$

and the Euler-Lagrange equations, Eqs. 10 and 11, become

$$\begin{aligned} &\dot{\lambda}_1 - \frac{\lambda_2}{v}\dot{\theta} + \lambda_3\sin\theta - \cos\theta = 0 \\ &\dot{\lambda}_2 - \lambda_1 g\cos\theta + \frac{\lambda_2}{v}g\sin\theta + \lambda_3 v\cos\theta + v\sin\theta = 0 \\ &\dot{\lambda}_3 = 0 \\ &\frac{T}{m}\left[\lambda_1\sin\alpha - \frac{\lambda_2}{v}\cos\alpha\right] = 0. \end{aligned} \tag{34}$$

During powered flight $\frac{T}{m} \neq 0$, so that upon differentiation Eq. 34_4 becomes

$$\dot{\lambda}_1 v\sin\alpha - \dot{\lambda}_2\cos\alpha + \lambda_1\dot{v}\sin\alpha + \lambda_1 v\dot{\alpha}\cos\alpha + \lambda_2\dot{\alpha}\sin\alpha = 0. \tag{35}$$

Combining Eqs. 33, 34 and 35 leads to

$$\lambda_2\dot{\varphi} + (\lambda_3\cos\varphi + \sin\varphi)v\sin\alpha = 0 \tag{36}$$

where

$$\varphi = \theta + \alpha.$$

If we set

$$\lambda_3 = -\tan\varphi \tag{37}$$

both Eqs. 34_3 and 36 are satisfied, since then

$$\dot{\varphi} = 0,$$

that is, φ = constant so that λ_3 = constant.

We recall that $\lambda_{2_0} = 0$ when $v_0 = 0$. This condition is met, for Eq. 34_1 may be written

$$v^2[\cos\theta - \lambda_3\sin\theta - \dot{\lambda}_1] = \lambda_2 g\cos\theta \tag{38}$$

whence $\lambda_{2_0} = 0$ when $v_0 = 0$, if $\theta_0 \neq \frac{\pi}{2}$. However, $\theta_0 = \frac{\pi}{2}$ is clearly of no interest.

During free flight Eq. 34 with 33 and the first integral Eq. 19 reduce to

$$\begin{aligned} v\dot{\lambda}_1 - \frac{\lambda_1}{v}g\dot{y} + 2\lambda_3\dot{y} - 2\dot{x} &= 0 \\ \dot{\lambda}_2\dot{y} + \lambda_2 g + v^2 &= 0 \end{aligned} \tag{39}$$

where

$$\begin{aligned} \dot{x} &= v\cos\theta = p_b \\ \dot{y} &= v\sin\theta = q_b - gt \end{aligned} \tag{40}$$

and (p_b, q_b) are the velocity components at burn-out. Conditions at burn-out will be denoted by subscript b and

t_f = time of free flight.

Upon integration and use of boundary conditions

$$\lambda_{1_f} = \lambda_{2_f} = 0$$

Eqs. 39 yield

$$\lambda_{1b} = \frac{2}{v_b}\left[\lambda_3 q_b - p_b - \tfrac{1}{2}\lambda_3 g t_f\right] t_f$$
$$\lambda_{2b} = \left[q_b + \frac{p_b^2}{q_b - g t_f}\right] t_f. \tag{41}$$

At burn-out Eq. 19 may be written

$$\lambda_{1b} g q_b + \frac{\lambda_{2b}}{v_b} g p_b - \lambda_3 v_b q_b + v_b p_b = 0. \tag{42}$$

Substitution of Eq. 41 in Eq. 42 permits solution for λ_3, that is,

$$\lambda_3 = \frac{p_b}{q_b - g t_f} \tag{43}$$

or

$$\tan \varphi = \frac{p_b}{g t_f - q_b}. \tag{44}$$

This is precisely the result obtained by more direct methods.[2,4]

NOTE ON NINE-POINT ANALOGUES OF LAPLACE'S EQUATION

BY

DONALD GREENSPAN [1]

Subject to the assumptions and definitions made in a previous paper (3),[2] the writer proved, using only elementary methods, that

$$-20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8 = 0 \quad (1)$$

was a "best" difference equation approximation for the Laplace equation by showing that no higher order approximation existed which utilized only the points (x_0, y_0), $(x_0 + h, y_0)$, $(x_0 + h, y_0 + h)$, $(x_0, y_0 + h)$, $(x_0 - h, y_0 + h)$, $(x_0 - h, y_0)$, $(x_0 - h, y_0 - h)$, $(x_0, y_0 - h)$, $(x_0 + h, y_0 - h)$. The object of this note is to demonstrate rather simply the further result that there does not even exist another essentially different nine-point analogue of the same order as that of (1).

In order to construct any seventh order approximation, it follows from Eq. 11 of (3) that all the following equations must be satisfied:

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 = \epsilon_1, \quad \epsilon_1 = o(h^9) \quad (2.1)$$

$$\alpha_1 - \alpha_3 + \alpha_5 - \alpha_6 - \alpha_7 + \alpha_8 = \epsilon_2, \quad \epsilon_2 = o(h^7) \quad (2.2)$$

$$\alpha_2 - \alpha_4 + \alpha_5 + \alpha_6 - \alpha_7 - \alpha_8 = \epsilon_3, \quad \epsilon_3 = o(h^7) \quad (2.3)$$

$$\alpha_5 - \alpha_6 + \alpha_7 - \alpha_8 = \epsilon_4, \quad \epsilon_4 = o(h^6) \quad (2.4)$$

$$-\alpha_1 + \alpha_2 - \alpha_3 + \alpha_4 = \epsilon_5, \quad \epsilon_5 = o(h^6) \quad (2.5)$$

$$-\alpha_1 + \alpha_3 + 2\alpha_5 - 2\alpha_6 - 2\alpha_7 + 2\alpha_8 = \epsilon_6, \quad \epsilon_6 = o(h^5) \quad (2.6)$$

$$\alpha_2 - \alpha_4 - 2\alpha_5 - 2\alpha_6 + 2\alpha_7 + 2\alpha_8 = \epsilon_7, \quad \epsilon_7 = o(h^5) \quad (2.7)$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - 4\alpha_5 - 4\alpha_6 - 4\alpha_7 - 4\alpha_8 = \epsilon_8, \quad \epsilon_8 = o(h^4) \quad (2.8)$$

$$\alpha_2 - \alpha_4 - 4\alpha_5 - 4\alpha_6 + 4\alpha_7 + 4\alpha_8 = \epsilon_9, \quad \epsilon_9 = o(h^3) \quad (2.9)$$

$$\alpha_1 - \alpha_3 - 4\alpha_5 + 4\alpha_6 + 4\alpha_7 - 4\alpha_8 = \epsilon_{10}, \quad \epsilon_{10} = o(h^3) \quad (2.10)$$

$$\alpha_2 - \alpha_4 + 8\alpha_5 + 8\alpha_6 - 8\alpha_7 - 8\alpha_8 = \epsilon_{11}, \quad \epsilon_{11} = o(h) \quad (2.11)$$

$$-\alpha_1 + \alpha_3 - 8\alpha_5 + 8\alpha_6 + 8\alpha_7 - 8\alpha_8 = \epsilon_{12}, \quad \epsilon_{12} = o(h) \quad (2.12)$$

It is easily seen by considering linear combinations that: $3\epsilon_9 - 5\epsilon_7 + 2\epsilon_3 = 0$, $3\epsilon_{10} + 5\epsilon_6 + 2\epsilon_2 = 0$, $3\epsilon_{11} + 7\epsilon_7 - 10\epsilon_3 = 0$, $3\epsilon_{12} + 7\epsilon_6 + 10\epsilon_2 = 0$. Hence the above system of equations is equivalent to a system which consists of these latter four equations and Eqs. 21. through 2.8.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Let us then solve Eqs. 2.1 through 2.8 in terms of α_0. This solution is:

$$\alpha_1 = \frac{\epsilon_2}{3} - \frac{\epsilon_6}{6} - \frac{\epsilon_5}{4} + \frac{\epsilon_1}{5} - \frac{\alpha_0}{5} + \frac{\epsilon_8}{20} \tag{3.1}$$

$$\alpha_2 = \frac{\epsilon_3}{3} + \frac{\epsilon_7}{6} + \frac{\epsilon_5}{4} + \frac{\epsilon_1}{5} - \frac{\alpha_0}{5} + \frac{\epsilon_8}{20} \tag{3.2}$$

$$\alpha_3 = -\frac{\epsilon_2}{3} + \frac{\epsilon_6}{6} - \frac{\epsilon_5}{4} + \frac{\epsilon_1}{5} - \frac{\alpha_0}{5} + \frac{\epsilon_8}{20} \tag{3.3}$$

$$\alpha_4 = -\frac{\epsilon_3}{3} - \frac{\epsilon_7}{6} + \frac{\epsilon_5}{4} + \frac{\epsilon_1}{5} - \frac{\alpha_0}{5} + \frac{\epsilon_8}{20} \tag{3.4}$$

$$\alpha_5 = \frac{\epsilon_4}{4} + \frac{\epsilon_2}{12} + \frac{\epsilon_6}{12} + \frac{\epsilon_3}{12} - \frac{\epsilon_7}{12} - \frac{\epsilon_8}{20} + \frac{\epsilon_1}{20} - \frac{\alpha_0}{20} \tag{3.5}$$

$$\alpha_6 = -\frac{\epsilon_4}{4} - \frac{\epsilon_2}{12} - \frac{\epsilon_6}{12} + \frac{\epsilon_3}{12} - \frac{\epsilon_7}{12} - \frac{\epsilon_8}{20} + \frac{\epsilon_1}{20} - \frac{\alpha_0}{20} \tag{3.6}$$

$$\alpha_7 = \frac{\epsilon_4}{4} - \frac{\epsilon_2}{12} - \frac{\epsilon_6}{12} - \frac{\epsilon_3}{12} + \frac{\epsilon_7}{12} - \frac{\epsilon_8}{20} + \frac{\epsilon_1}{20} - \frac{\alpha_0}{20} \tag{3.7}$$

$$\alpha_8 = -\frac{\epsilon_4}{4} + \frac{\epsilon_2}{12} + \frac{\epsilon_6}{12} - \frac{\epsilon_3}{12} + \frac{\epsilon_7}{12} - \frac{\epsilon_8}{20} + \frac{\epsilon_1}{20} - \frac{\alpha_0}{20} \tag{3.8}$$

Hence, Eq. 9 of (3), that is,

$$L_9(u) = \sum_0^8 \alpha_i u_i, \tag{4}$$

becomes by substitution of Eqs. 3.1 through 3.8, and by recombination:

$$\begin{aligned} L_9(u) = &-\frac{\alpha_0}{20}[-20u_0+4(u_1+u_2+u_3+u_4)+u_5+u_6+u_7+u_8] \\ &+\frac{\epsilon_1}{20}[4(u_1+u_2+u_3+u_4)+u_5+u_6+u_7+u_8]+\frac{\epsilon_2}{12}[4u_1-4u_3+u_5-u_6-u_7+u_8] \\ &+\frac{\epsilon_3}{12}[4u_2-4u_4+u_5+u_6-u_7-u_8]+\frac{\epsilon_4}{4}[u_5-u_6+u_7-u_8] \\ &+\frac{\epsilon_5}{4}[-u_1+u_2-u_3+u_4]+\frac{\epsilon_6}{12}[-2u_1+2u_3+u_5-u_6-u_7+u_8] \\ &+\frac{\epsilon_7}{12}[2u_2-2u_4-u_5-u_6+u_7+u_8]+\frac{\epsilon_8}{20}[u_1+u_2+u_3+u_4-u_5-u_6-u_7-u_8]. \end{aligned} \tag{5}$$

Substitution of Taylor series for u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8 about (x_0, y_0), and recalling the various orders of magnitude of the ϵ_i, $i = 1, 2, 3, 4, 5, 6, 7, 8$, yields after much unexciting calculation, that:

$$\frac{\epsilon_1}{20}[4(u_1+u_2+u_3+u_4)+u_5+u_6+u_7+u_8] = o(h^6) \tag{6.1}$$

$$\frac{\epsilon_2}{12}[4u_1-4u_3+u_5-u_6-u_7+u_8] = o(h^6) \tag{6.2}$$

$$\frac{\epsilon_3}{12}[4u_2-4u_4+u_5+u_6-u_7-u_8] = o(h^6) \tag{6.3}$$

$$\frac{\epsilon_4}{4}[u_5-u_6+u_7-u_8] = o(h^6) \tag{6.4}$$

$$\frac{\epsilon_5}{4}[-u_1+u_2-u_3+u_4] = o(h^5) \tag{6.5}$$

$$\frac{\epsilon_6}{12}[-2u_1+2u_2+u_5-u_6-u_7+u_8] = o(h^5) \tag{6.6}$$

$$\frac{\epsilon_7}{12}[2u_2-2u_4-u_5-u_6+u_7+u_8] = o(h^5) \tag{6.7}$$

$$\frac{\epsilon_8}{20}[u_1+u_2+u_3+u_4-u_5-u_6-u_7-u_8] = \frac{\epsilon_8}{20}[2h^2(A_{2,0}+A_{0,2})-4h^2(A_{2,0}+A_{0,2})+o(h^4)]$$

$$= \frac{\epsilon_8}{20}\cdot o(h^4) = o(h^5). \tag{6.8}$$

This last result follows since: $A_{2,0} + A_{0,2} = 0$. Hence, by Eqs. 6.1 through 6.8, Eq. 5 becomes

$$L_9(u) = -\frac{\alpha_0}{20}[-20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8] + o(h^5). \tag{7}$$

This result is valid independent of the choices of the ϵ_i, $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$, subject of course to the necessary restrictions on their orders of magnitude. Hence the only possible seventh order approximation results by discarding the terms $o(h^5)$ in Eq. 7 and by setting the difference expression equal to zero, that is,

$$-\frac{\alpha_0}{20}[-20u_0 + 4(u_1 + u_2 + u_3 + u_4) + u_5 + u_6 + u_7 + u_8] = 0.$$

If $\alpha_0 = 0$, no difference equation results. If $\alpha_0 \neq 0$, then Eq. 1 is the unique seventh order difference approximation, where of course one means unique to within a multiplicative constant.

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(5) W. E. MILNE, "Numerical Solution of Differential Equations," New York, John Wiley & Sons, Inc., 1953.

New Giant Brain for Businessmen. —The first Datamatic 1000, a new $2½-million giant "brain," successfully completed its first public demonstrations recently. A critical audience of over 100 businessmen, government and municipal officials and executives watched the large-scale computer system built by Datamatic Corp. Newton Highlands, Mass. whiz through a full day of demonstrations on sample business problems. It exhibited an elephantine memory, and showed it could "read" and "write" at eye-blinking speeds.

The new computer system, which occupies 5000 sq. ft., is the latest development in the explosive evolution of electronic brains since World War II's Mark I scientific model. It is said to be the first designed exclusively for business use. Its builder, the two-year-old Datamatic Corp., is a firm originally formed by Minneapolis-Honeywell and Raytheon. Honeywell, senior partner, subsequently bought out Raytheon for $4½ million and is now sole owner.

The system was shipped to Detroit where it is being installed at the Michigan Hospital Service, the nation's second-largest Blue Cross-Blue-Shield service. There it will watch over the personal statistics of 3½ million residents, processing an average of 25,000 record changes per day in less than two hours.

To businessmen, the principal attraction of the $2¼-to-$2½-million system is its speed and capacity in a balanced system, according to Walter W. Finke, President of Datamatic. It is the first computer system with the ability to locate, use and return information to magnetic tape files as fast as its high-speed internal section can accept and process it. One customer, Los Angeles County, estimates that this ability will save the County $500,000 annually, over and above the cost of the system.

He cited some of the "brain's" vital statistics. It can "read" and "write" simultaneously at the rate of 60,000 digits a second, at the same time handling 4000 additions, 1000 multiplications or 5000 comparisons. Its central brain can search through the equivalent of 7500 punched cards a second, or 4 million punched cards in less than ten minutes. Each reel of its magnetic tape (3 in. wide; 31 channels of information) can hold data equal to that found on 465,000 punched cards. A full system, incorporating 100 magnetic tape units, can absorb 46½ million punched cards worth of information.

Answers, or reports, are turned out at the rate of 900 printed lines a minute or 60,000 punched cards an hour.

Finke said that this ability to handle a tremendous amount of data quickly overcomes what has been an inherent weakness in the early giant brains. These brains, he explained, were designed by scientists to handle a limited amount of data, do a lot of complicated calculation and get a concise answer.

"But business has huge amounts of paperwork which it wants to feed into a computer quickly and get all the answers as fast as possible. Involved is only a minimum amount of computation. That is what this new computer system has been expressly designed to do," he said.

THE MINIMALITY OF RECTIFIER NETS WITH MULTIPLE OUTPUTS INCOMPLETELY SPECIFIED *

BY

ROBERT McNAUGHTON [1] AND BERNON MITCHELL [2]

The problem of finding a minimal rectifier net (for use in a computing machine) with one output realizing a given truth function is merely the problem of finding a simplest normal equivalent formula as studied by Quine and others. The key concept is that of a *prime implicant.* The problem is more difficult if the truth function as given is incompletely specified; in other words, there are certain input combinations for which the output can either be in state 1 or state 0 (true or false) indifferently. This problem is known as the "don't care" problem. It is still more difficult when a net with multiple outputs is required, because then it is often possible for a fundamental formula to be used simultaneously in the formulas representing many outputs; thus it is not possible to find the minimal net simply by minimizing the formula for each of the outputs in turn. The main contribution of this paper is the extension to this problem of the concept of prime implicant: an r^{th} *order prime implicant* for r outputs is, roughly, a fundamental formula suitable to appear in the formulas for these r outputs in a minimal solution. Our problem is fully general, except that we assume through most of the paper that the various outputs are specified for exactly the same input combinations, though, of course, not always specified in the same way.

The first two sections will serve to introduce the reader to rectifier nets. Those who are familiar with them may omit these two sections with the exception of the last paragraph of Section 1. The third section states the problem. The fourth section contains the definition of r^{th} order prime implicant and theorems which justify the concept. Section 5 contains an heuristic explanation of Section 4. Section 6, finally, contains the solution and discussion of a sample problem and many informal suggestions as to the general procedure to follow both in finding the prime implicants and in selecting from them fundamental formulas for a minimal solution.

It came to our attention in the late stages of the preparation of this paper that David E. Muller had shown how to reduce the multiple output problem to the single output problem, for both synthesis and

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minimization.[3] His method is applicable to rectifier nets. The question is, is the method adapted from his paper or ours more practical for finding a minimal solution? We do not know the answer but suspect that for certain problems his method is superior and for other problems our method is superior.

1. RECTIFIER NETS: GENERAL DESCRIPTION

A computer engineer often finds himself with the task of designing an electronic circuit performing an involved logical operation, as part of a large scale digital computer. This circuit might be thought of as the inside of a black box with n input wires and m output wires, where each of the wires has one of two possible states, say, positive with respect to ground potential or at ground potential. The state of each of the m output wires will be considered as being defined by some logical function of the combination of states on the n input wires. The black box we have in mind has no memory, that is, no parts which function as useful delays. The states of the outputs at any instant of

TABLE I.

p	q	r	ϕ_1	ϕ_2	ϕ_3
1	1	1	1	1	0
1	1	0	1	0	1
1	0	1	1	0	0
1	0	0	0	1	1
0	1	1	1	1	0
0	1	0	0	0	1
0	0	1	0	0	0
0	0	0	0	1	1

time are therefore determined solely by the states of the inputs at approximately that instant of time. Truth tables notation can be used conveniently to describe the states of these output wires as a function of the input wires. These truth tables contain, instead of T's and F's representing truth and falsity, 1's and 0's which represent, respectively, positive potential and ground potential of a wire.

For example, consider Table I in which $m = n = 3$. Here the states of the three input wires are represented by the three independent variables p, q, and r. The states of the three output wires are represented by the three dependent variables ϕ_1, ϕ_2, and ϕ_3. As can be seen from the first row of the table, when p, q, and r all take on the value 1, then ϕ_1, ϕ_2, and ϕ_3 take on the values 1, 1, and 0, respectively. If we look only at the entries under p, q, and r and read these entries as a three-digit binary number, the rows count down from seven to zero. For example, 1 0 1 is the binary representation of $5 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$. From this it can be seen that the eight rows exhaust all

[3] DAVID E. MULLER, "IRE Transactions on Electronic Computers," Sept., 1954.

possible combinations of input states for three wires. The number of rows required for a truth table with n independent input wires will be 2^n. The number of distinct functions that can exist for a black box with n input wires and a single output wire is $2^{(2^n)}$. The number of distinct sets of m functions that can exist with n input wires and m output wires is $[2^{(2^n)}]^m$; many of these, however, will be quite useless.

There are many types of electronic hardware that might be used in the box (relay, tubes, rectifiers, etc.) to meet the requirements of a given truth table. This paper will be concerned only with boxes constructed from rectifiers, wires and resistors. When the truth table for some black box problem of this kind is given it is an easy matter to write out in the sentential calculus formulas for the ϕ's containing the input variables (p, q, r, etc.). For example, the outputs of the above table can be represented as developed disjunctive normal formulas (see the end of this section where "disjunctive normal formula" and some

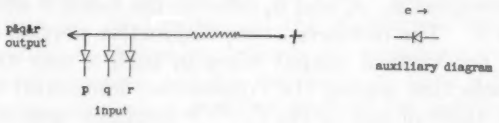


FIG. 1.

other terms used in this section are defined) as is evident from the following equivalences:

$$\phi_1 \equiv pqr \vee pq\bar{r} \vee p\bar{q}r \vee \bar{p}qr$$

$$\phi_2 \equiv pqr \vee p\bar{q}\bar{r} \vee \bar{p}qr \vee \bar{p}\bar{q}\bar{r}$$

$$\phi_3 \equiv pq\bar{r} \vee p\bar{q}\bar{r} \vee \bar{p}q\bar{r} \vee \bar{p}\bar{q}\bar{r}$$

In this notation the conjunction sign "&" between the small letters is suppressed; "v" is used for disjunction and bars over the small letters are used for negation. (For the readers familiar with the notation of Boolean algebra used by many circuit engineers, the truth functional formulas of this paper need not appear strange. The wedge, "v" can be thought of simply as another version of the plus, +, meaning Boolean union, and the equivalence sign, ≡, as the equal sign =.) For each disjunct (which is itself a conjunction) in the above formulas, there corresponds a rectifier circuit which realizes it (that is, which has that conjunction as an output). For example, $p\&q\&r$ is realized as shown in Fig. 1. In this diagram some constant positive voltage is applied to the point in the diagram with a "+" by it. The convention in this paper for electron flow through the rectifiers is given in the auxiliary diagram.

There also exist well-known rectifier circuits to express a disjunction

of disjuncts. For example, to realize ϕ_1, see Fig. 2. Combining this type of "or" circuit with the previous type of "and" circuit, ϕ_1, ϕ_2, and ϕ_3 may be expressed in the circuit shown in Fig. 3. In this rectifier net it is to be understood that the only electrical connections between the vertical and the horizontal wires are through the rectifiers and resistors. The first six horizontal wires in the conjunctive portion of the rectifier net correspond to the first six rows in Table I and the seventh horizontal wire corresponds to the eighth row of the table.

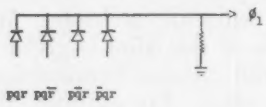


FIG. 2.

There is no correspondent to the seventh row of the table since all of the output functions ϕ_1, ϕ_2 and ϕ_3 take on the value 0 when the inputs $\bar{p}$, $\bar{q}$ and r are 1. The rectifiers in the disjunctive portion of the net are placed along the vertical output wires in such a way as to make an appropriate selection among the conjunctive horizontal wires. Thus, given a truth table of any of the $[2^{(2^n)}]^m$ available sets of m functions it is possible to construct, at least in theory, a rectifier net which expresses it. It is only necessary to express each of the output functions

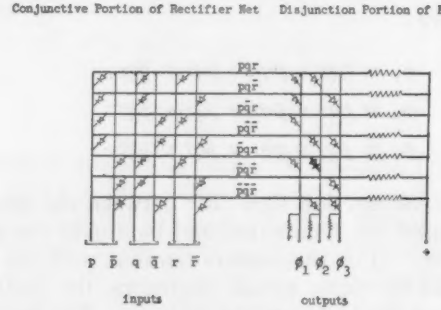


FIG. 3.

in developed disjunctive normal form and then to build the rectifier net which corresponds to them. We note that, if m and n are too large, it may not be feasible electrically to construct the kind of net we discuss in this paper. The practical limitations on the size of m and n seem to be dictated by the failure of real diode rectifiers to meet the specification of the idealized diode (that is, a diode with zero forward resistance, infinite backward resistance and no limitation on current carrying capacity).

Another engineering consideration is that the resistors along the horizontal wires will have to be adjusted to give equal voltage on the output wires when they are positive. The resistance of a resistor will depend in part on the number of rectifiers attached to the corresponding horizontal wire in the disjunctive portion of the net.

It will be noticed here that the above net deviates from the original black box model in that instead of three input wires there are three partitioned pairs of input wires. It is indicated in the notation of the net diagram that one and only one input wire of a partitioned pair may be in the positive state at a given instant. The rectifier net could be made to conform to the original model by adding some auxiliary apparatus (double throw relay or a "flip-flop" circuit, for instance) to produce a partitioned pair of input wires from a single input wire.

It might be mentioned here that, if the designer of a rectifier net desires a negative potential output with respect to ground instead of positive, there exists a similar net for this. To do this conjunctive normal formulas are used instead of the disjunctive normal formulas. We shall confine ourselves exclusively to problems concerning disjunctive normal formulas. If it is necessary to work with conjunctive normal formulas, there is a familiar method for converting it into a problem concerning disjunctive normal formulas.

In the remainder of the paper when we speak of rectifier nets we shall mean a circuit such as the one in Fig. 3. The important features of such nets for us are the following: (1) the net has two portions, a conjunctive portion and a disjunctive portion; (2) the input pair wires (which we call *inputs*) are in the conjunctive portion, and the output wires (which we call *outputs*) are in the disjunctive portion; (3) there are intermediate wires (indicated by horizontal lines in figures of this paper) each of which connects some of the inputs to some of the outputs; (4) if an intermediate wire is connected to several inputs, then each of these connections must involve a rectifier; if to only one input, a wire link without a rectifier is used; (5) if an output is connected to several intermediate wires, each of the connections must involve a rectifier; if to only one intermediate wire, a wire link without a rectifier is used ((4) and (5) will be explained in the next section); (6) there are no rectifiers in the net other than those described in (4) and (5); (7) no output is in state 1 and no output is in state 0, for every combination of inputs. (For such an output could be connected directly to positive voltage or to ground and is therefore trivial. For logical considerations it is easier to exclude such cases.)

We shall now define some logical terms common in the literature. A *literal* is either a letter itself or a letter with a bar over it (symbolizing an input). A *fundamental formula* (abbreviated: FF) is either a literal or a conjunction of literals without any repetitious letter. Thus $\bar{p}$, and $pqrs$ are FF but $pq\bar{p}r$ and $pq\bar{p}s$ are not FF. A *normal formula* (or

disjunctive normal formula) is either an FF or a disjunction of FF. A *developed normal formula* is a normal formula every FF of which contains every letter that the formula itself contains; we say the formula is developed with respect to those letters. Thus $\bar{p}q$ and $pqr \vee p\bar{q}r \vee \bar{p}qr$ are developed normal formulas, although they are developed with respect to different sets of letters. $\bar{p}q \vee ps \vee qst$ is a normal formula which is not developed.

2. THE COST OF A RECTIFIER NET

Expressing each output function of a rectifier net directly in terms of the developed normal formula easily shows that there exists a net that corresponds to any possible set of m output functions. However, this method in general makes an extravagant use of rectifiers. It is usually possible through the use of normal formulas which are not developed to find a simpler net. The saving in general is enough to justify considering in detail a method for finding a simplest net expressing given output functions. To see to what extent simplification

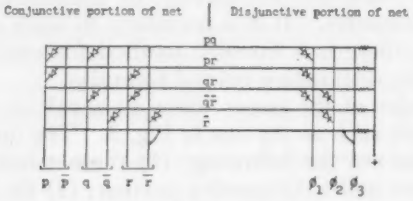


FIG. 4.

can be made, let us take the functions ϕ_1, ϕ_2, ϕ_3 described in Section 1. By means of the sentential calculus the following equivalences can be verified.

$$\phi_1 \equiv pq \vee pr \vee qr$$
$$\phi_2 \equiv qr \vee \bar{q}\bar{r}$$
$$\phi_3 \equiv \bar{r}.$$

Converting these functions into a disjunctive rectifier net gives a net shown in Fig. 4, which is obviously simpler than the net shown in Fig. 3, although it realizes the same set of output functions.

The lack of rectifiers on the lowest intermediate wire of Fig. 3 is in accord with the description of a net at the end of Section 1. When there is more than one connection to an output, the substitution of a wire link on this output introduces the possibility of a short circuit for certain input combinations. Similarly, when more than one connection to an intermediate wire in the conjunctive portion of the net is required, the substitution of a wire link on this intermediate wire also leads to

short circuits. But, in each case, if there is only one such connection there is no possibility of a short circuit.

Whether a connection in a net requires a rectifier can be seen by a simple inspection of the formulas used in the construction of the net. If the normal formula for an output function contains no disjunction signs (that is, "v"), then it is possible to make the substitution of a wire link for a rectifier on the corresponding output wire. For example, if $\phi_1 = pq$, then this would require two rectifiers on the conjunctive portion of the net and only a wire link for the output in the disjunctive portion as shown in Fig. 5. Similarly, if the normal formula for an

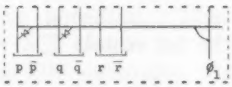


FIG. 5.

output function contains a disjunct of just a single letter, or the letter's negation, then a wire link may be substituted for the rectifier on the conjunctive portion of the net. For example, if $\phi_1 = r \vee pq$, we may make the wire link substitution as shown in Fig. 6.

It is clear that for a given problem, if minimality is disregarded, there is always more than one solution. That is to say, for a given set of output functions there is more than one net whose outputs realize those functions. A net will be better than another if it costs less, where the cost must include, among other factors, price of parts and labor used in the construction.

It is our purpose to devise a general minimization method for such circuits and since the cost of such nets depends largely on the special

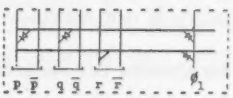


FIG. 6.

circumstances of the manufacturer, we find that it is necessary to define a cost function of a net which merely approximates the actual cost. We therefore define the cost of a net to be equal to the number of rectifiers used in the net. Our reasons are, first, that rectifiers are more expensive than the wires and resistors. Second, the number of man-hours used to construct the net is roughly proportional to the number of rectifiers. Having thus defined the cost of a net we find that our problem of minimization is no longer an engineering problem but a problem in pure logic. One can determine the design of a minimal net by considering only the formulas representing the outputs. This procedure has a great advantage in view of the fact that the problem

so conceived is manageable, and, in many applications, justifies our cost function.

We give a method of computing the cost of a net by examining the formulas representing the outputs. Suppose there are m outputs, and suppose that the formula ϕ_i corresponds to the i^{th} output for each i, $1 \leq i \leq m$. Let $c_i = 0$ if there is one FF in ϕ_i; otherwise, let $c_i =$ the number of FF in ϕ_i. (In short, c_i is the number of disjuncts of ϕ_i.) Let the FF of all the formulas $\phi_1, \ldots, \phi_m$ be, without repetition, $\alpha_1, \ldots, \alpha_p$. Let $d_i = 0$ $(1 \leq i \leq p)$ if there is one literal in α_i; otherwise let d_i be the number of literals of α_i. (In short, d_i is the number of conjuncts of α_i.)

Theorem. The cost of the net equals $\sum_{i=1}^{m} c_i + \sum_{i=1}^{p} d_i$.

Proof: By remarks above $\sum_{i=1}^{m} c_i$ is the number of rectifiers in the disjunctive portion of the net, and $\sum_{i=1}^{p} d_i$ is the number of rectifiers in the conjunctive portion of the net. (Note that we do not count an FF more than once when it appears in more than one output formula, since only one intermediate wire for it is required in the net.)

From the description of the net at the end of the first section it is clear that if all the intermediate wires of a net are represented by FF, then the outputs are represented by normal formulas. It is also clear that an intermediate wire is represented by a conjunction of literals or a single literal. In Section 4 we shall show that in restricting these conjunctions to FF minimality is not sacrificed.

3. BROADENING THE PROBLEM

Since a rectifier net can be constructed from the output formulas which are, in effect, a recipe for the net, and since the cost of the net is easily computed from those formulas, we shall assume that the solution to the problem of synthesis and minimization of such nets involves only the output formulas. This problem is rather general in that there may be any number of inputs and any number of outputs. We make the problem even more general by allowing for the possibility that the truth tables by means of which the outputs are given contain besides 0's and 1's some blank spaces. Such a blank space will mean that as far as the problem is concerned we don't care whether the output is in state 0 or state 1 for that input combination; such input combinations are called *unspecified cases*; other input combinations, those for which we do care, are called *specified cases*. The problem of minimization increases in difficulty as the number of unspecified cases

increases (up to a certain point); the reason lies in the increase in the number of nets realizing the output functions among which the search for the minimal has to be made. In this paper we are concerned with a method of minimization appropriate to problems in which there is an arbitrary number of unspecified cases. However, our results are limited to problems in which the unspecified cases are the same for all outputs. Some of our results can be extended to the more general problem where different outputs have different unspecified cases, as noted at the end of Section 5.

Another restriction we shall make is that each output as given is specified to be 1 for at least one input combination and 0 in at least one input combination. Otherwise, it could be simply connected to ground or the source of positive potential. This restriction simplifies the logical considerations.

Consider a rather simple example. Given an incomplete truth table (Table II) for a rectifier net with $n = 2$ and $m = 1$: What truth value assignment should be made to the blank space in the second row to minimize the cost of the net? If a 1 is used, $\phi_1 = p \vee q$ with a net cost of 2. If 0 is used, $\phi_1 = q$ with a net cost of 0. When n, m and the

TABLE II.

p	q	ϕ_1
1	1	1
1	0	
0	1	1
0	0	0

number of unspecified cases are small, it is sometimes possible to minimize the cost of the net with a few good guesses. We propose a method which seems practicable for reasonably large m and n. We shall show later a problem with $n = 5$, $m = 4$ and the number of unspecified cases equal to 22 that was solved in a few hours with pencil and paper. (The aid of an electronic calculator seems feasible with larger m and n.)

We note that the "don't care problem" (that is, the problem with some unspecified cases) so conceived is the same as the problem of constructing a net in which it is given that certain input combinations do not arise (for electrical reasons having to do with what is outside our net). This fact is often stated loosely, but we find that the somewhat detailed explanation offered in the following paragraph is necessary.

We must investigate whether our cost function still applies if the unspecified cases never occur. Use of the cost function assumes that a wire link is to be substituted for a rectifier when and only when a single connection to an output wire is required or when a single connection to an intermediate wire in the conjunctive portion of the net is required. It might be thought that if the unspecified cases never occur,

wire links could be substituted for other rectifiers without the danger of a shorting of any rectifiers which remain. Suppose, for example, that the FF pqr was used in a net of minimum cost and further that the unspecified cases are such that r never takes on the value 1, while p or q is 0. Then the rectifiers that serve p and q would not be shorted if a wire link were substituted for r. Such a substitution would reduce the cost of the net by 1, and hence one might doubt the correctness of the given cost function for such problems. It can be shown, however, that this situation will in fact never arise in a minimal solution. It will be noticed that the FF pqr takes on the value 1 when and only when r takes on the value 1. This means that r may be used in place of pqr (whether or not the unspecified cases ever occur). Hence pqr would not be in a net of minimum cost such as given in the example. Similar considerations restrict the substitution of wire links on the vertical output wires in nets of minimum cost. Thus our problem is the same (and the cost function is equally appropriate) for nets in which the unspecified cases never occur, as for nets in which the unspecified cases do occur.

4. PRIME IMPLICANTS

Suppose that a truth table is given for the m outputs as incompletely specified functions of the inputs. For each i from 1 to m, let $\psi_T{}^i$ be a formula which has the value 1 when ϕ_i is specified to be 1, and has the value 0 otherwise; let $\psi_F{}^i$ be a formula which has the value 1 when ϕ_i is specified to be 0, and has the value 0 otherwise. For example, $\psi_T{}^i$ and $\psi_F{}^i$ may be developed normal formulas, whereupon the disjuncts of $\psi_T{}^i$ would correspond to the rows of the truth table in which ϕ_i has the value 1 or true (hence the subscript T), and the disjuncts of $\psi_F{}^i$ to the rows in which ϕ_i is 0 or false (hence the subscript F). Recall that the unspecified cases are the same for all the outputs: Let U be a formula which has the value 1 in the unspecified cases and the value 0 in the specified cases.

We recall from the logical literature that a formula is *valid* if it has the value 1 in all rows of the truth table, *inconsistent* if 0 in all rows, and *consistent* if 1 in at least one row; a formula χ_1 *implies* χ_2 if $\chi_1 \rightarrow \chi_2$ is valid. $\rightarrow$ is the sign for material implication; thus $\chi_1 \rightarrow \chi_2$ is equivalent to $\bar{\chi}_1 \vee \chi_2$. $\equiv$ is the sign for material equivalence; $\chi_1 \equiv \chi_2$ is equivalent to $(\chi_1 \rightarrow \chi_2) \,\&\, (\chi_2 \rightarrow \chi_1)$.

Quine uses the concept of prime implicant for a problem which can be regarded as a special case of our problem, namely the case where there is exactly one output and no unspecified cases. A prime implicant, in that problem, is an FF which implies the output function but implies no shorter FF which implies the output function. Perhaps the main contribution of this paper is the extension of this concept for the more general problem. I is an r^{th} *order prime implicant* for the output functions $\phi_{i_1}, \phi_{i_2}, \ldots, \phi_{i_r}$ if the following conditions hold.

1. I is an FF.
2. I & $(\psi_{F^{i_1}} \vee \psi_{F^{i_2}} \vee \ldots \vee \psi_{F^{i_r}})$ is inconsistent.
3. I & $(\psi_{T^{i_1}} \& \psi_{T^{i_2}} \& \ldots \& \psi_{T^{i_r}})$ is consistent.
4. There does not exist an I' satisfying Conditions 1 through 3 such that
   a. the number of literals in I' is less than the number of literals in I;
   b. $(\psi_{T^{i_1}} \& \psi_{T^{i_2}} \& \ldots \& \psi_{T^{i_r}}) \rightarrow (I \rightarrow I')$ is valid.
5. There does not exist an I' satisfying Conditions 1 through 3 such that
   a. the number of literals in I' equals the number of literals in I;
   b. $(\psi_{T^{i_1}} \& \psi_{T^{i_2}} \& \ldots \& \psi_{T^{i_r}}) \rightarrow (I \rightarrow I')$ is valid;
   c. $(\psi_{T^{i_1}} \& \ldots \& \psi_{T^{i_r}}) \rightarrow (I' \equiv I)$ is not valid.

These conditions and the theorems of this section are informally explained and heuristically discussed in Section 5.

Note that the r^{th} order prime implicants for $\phi_{i_1}, \ldots, \phi_{i_r}$ are uniquely determined given the three truth functions U, $\psi_{T^{i_1}} \& \ldots \& \psi_{T^{i_r}}$, and $\psi_{F^{i_1}} \vee \ldots \vee \psi_{F^{i_r}}$. Furthermore, since $\psi_{F^{i_1}} \vee \ldots \vee \psi_{F^{i_r}}$ is equivalent to $- U \& - (\psi_{T^{i_1}} \& \ldots \& \psi_{T^{i_r}})$ and U is fixed in any given rectifier net problem, the r^{th} order prime implicants for $\phi_{i_1}, \ldots, \phi_{i_r}$ are uniquely determined given the single truth function $\psi_{T^{i_1}} \& \ldots \& \psi_{T^{i_r}}$. This fact proves useful in the search for prime implicants, as we shall see in Section 6.

To show the usefulness of the concept of an r^{th} order prime implicant, we show in the theorems which follow that prime implicants may always be used in constructing a net of minimal cost. Recall that a problem is given as a truth table in which each output is 0, 1, or unspecified in each row, and in which all the inputs have their unspecified cases in common. A set of formulas for $\phi_1, \ldots, \phi_m$ is a *solution* if, for each i, ϕ_i has the value 0 when the i^{th} output is specified to be 0 and the value 1 when it is specified to be 1, and if there is a net constructible from these formulas. A set of formulas is a *minimal solution* if it is a solution and if the cost of the corresponding net is no greater than any other solution.

Theorem 4.1. A minimal solution contains only normal formulas.

Proof: By a remark in Section 2, it is necessary only to show that all conjunctions of literals used in a minimal solution are FF. A conjunction of literals which is not an FF has a repeated letter. We show that any such conjunction can either be deleted or replaced by a shorter conjunction and the result is equivalent to the original formula. From this it follows that a minimal solution cannot contain such a conjunction.

Case I. The repeated letter appears in the conjunction both with a bar, and without, for example, $p\bar{p}q$. This conjunction always has the value 0, and so, by virtue of the stipulation that output functions are specified as 1 for some input combinations, this conjunction is not alone but appears as a disjunct in a disjunction. When it is deleted the resulting formula is equivalent to the original.

Case II. The conjunction has a repeated literal, for example, ppq or $pppq\bar{r}$. If all occurrences of the literal but one are deleted, the resulting formula is equivalent to the original. (The meaning of such a conjunction in a rectifier net has never been explained. Nor need it be.)

Theorem 4.2. An FF I appearing in the output formulas for $\phi_{i_1}, \ldots, \phi_{i_r}$ in a solution satisfies Condition 2 in the definition of an r^{th} order prime implicant for $\phi_{i_1}, \ldots \phi_{i_r}$.

Proof: Whenever I has the value 1, $\phi_{i_1}, \ldots, \phi_{i_r}$ will all have the value 1, since for each ϕ_{i_k}, I either is ϕ_{i_k} or is a disjunct of it. If $I \,\&\, (\psi_F{}^{i_1} \vee \ldots \vee \psi_F{}^{i_r})$ were consistent, contrary to Condition 2, then for some i_k, $I \,\&\, \psi_F{}^{i_k}$ would be consistent. Thus there would be a truth table row in which both I and $\psi_F{}^{i_k}$ had the value 1. For such a row, ϕ_{i_k} would be specified as 0 but would actually have the value 1 by virtue of I. Hence, Condition 2 must be satisfied.

Theorem 4.3. An FF I appearing in the formulas $\phi_{i_1}, \ldots, \phi_{i_r}$ and no others in a minimal solution satisfies Conditions 3 and 4 in the definition of r^{th} order prime implicant for $\phi_{i_1}, \ldots, \phi_{i_r}$.

Proof for Condition 3. A minimal solution is a solution and so, by Theorem 4.2, I implies $-(\psi_F{}^{i_1} \vee \ldots \vee \psi_F{}^{i_r})$ which is equivalent to $-\psi_F{}^{i_1} \,\&\, \ldots \,\&\, -\psi_F{}^{i_r}$. Since $-\psi_F{}^{i_j}$ is equivalent to $U \vee \psi_T{}^{i_j}$, for each j, the last formula of the preceding sentence is equivalent to $(U \vee \psi_T{}^{i_1}) \,\&\, (U \vee \psi_T{}^{i_2}) \,\&\, \ldots \,\&\, (U \vee \psi_T{}^{i_r})$ and therefore to $U \vee (\psi_T{}^{i_1} \,\&\, \ldots \,\&\, \psi_T{}^{i_r})$. Thus, if $I \,\&\, (\psi_T{}^{i_1} \,\&\, \ldots \,\&\, \psi_T{}^{i_r})$ were inconsistent, I would imply U; I would have the value 0 in all the specified cases. Hence, where I appears it would appear as a disjunct and not as a whole formula. I then could be deleted as a disjunct in any formula for any of the outputs without affecting any of the specified cases. Hence, I would not appear in a minimal solution.

Proof for Condition 4. Suppose Condition 4 is not satisfied for I; in other words, that some I' satisfies (a) and (b) of Condition 4. For any j between 1 and r, the solution for the $i_j{}^{\text{th}}$ output is either $\chi_{i_j} \vee I$, where χ_{i_j} is a normal formula, or I. Suppose first that it is the former. $\chi_{i_j} \vee I'$ will have the same truth value as $\chi_{i_j} \vee I$ in all the specified cases, which can be seen as follows. In a specified case where $\chi_{i_j} \vee I$ is 0, χ_{i_j} and I are each 0 and $\psi_F{}^{i_j}$ is 1; since I' satisfies Condition 2, I' is 0; and so, therefore, is $\chi_{i_j} \vee I'$. In a specified case where $\chi_{i_j} \vee I$ is 1, χ_{i_j} may be 1, whence $\chi_{i_j} \vee I'$ is 1. Or if χ_{i_j} is not 1, I is 1; since I satisfies Condition 2 (otherwise it could not appear in the solution as was to be proved) none of the $\psi_F{}^{i_1}, \ldots, \psi_F{}^{i_r}$ is 1 and hence $\psi_T{}^{i_1} \,\&$

. . . & $\psi_{T^{i_r}}$ is 1; by Condition 4 (*b*), I' is 1 and so is $\chi_{i_j} \vee I'$. This completes the proof that for each j, $1 \leq j \leq r$, if the i_j^{th} formula is $\chi_{i_j} \vee I$, I is replaceable by I' in the solution. If the i_j^{th} formula is I the proof that I' can be used instead is similar. Thus if I is replaced by I' in all r formulas, the cost will be decreased since I' has fewer literals.

Theorem 4.4. There exists a minimal solution in which every FF appearing in the formulas $\phi_{i_1}, \ldots, \phi_{i_r}$, and no others, satisfies Condition 5 in the definition of r^{th} order prime implicant for $\phi_{i_1}, \ldots, \phi_{i_r}$.

Proof: Suppose there is a minimal solution in which a fundamental formula I_1 is a disjunct in each of the formulas $\phi_{i_1}, \ldots, \phi_{i_r}$ and no others and there exists (contrary to Condition 5) an I_2 which is related to I_1 as I' is to I as stated in Condition 5. Indeed, suppose there is a sequence $I_1, I_2, I_3, \ldots$ such that for each i, if I_{i+1} is in the sequence it is related to I_i as the I' of Condition 5 is related to I. There must be a last member of this sequence I_q such that there is no I_{q+1} related to it as I' is to I, which can be seen as follows: Conditions 5(*b*) and 5(*c*) imply that the set of specified cases in which $\phi_{i_1}, \ldots, \phi_{i_r}$ and I are all in state 1 is a proper subset of the set of all specified cases in which $\phi_{i_1}, \ldots, \phi_{i_r}$ and I' are all in state 1; since there are only finitely many specified cases in which $\phi_{i_1}, \ldots, \phi_{i_r}$ are in state 1, the sequence $I_1, I_2, \ldots$ must be finitely long. Furthermore, it is not difficult to see that each member I_i of the sequence is related to each preceding member I_j as I' is to I. And so I_q is related to I_1 as I' is to I, but nothing is so related to I_q. I_q then satisfies Conditions 1, 2, 3 and 5. Furthermore, (*a*) the number of literals of I_q equals the number of literals of I_1; (*b*) $\psi_{T^{i_1}}$ & . . . & $\psi_{T^{i_r}} \rightarrow (I_1 \rightarrow I_q)$ is valid; and (*c*) $\psi_{T^{i_1}}$ & . . . & $\psi_{T^{i_r}} \rightarrow (I_q \equiv I_1)$ is not valid. In the minimal solution in which I_1 appears in $\phi_{i_1}, \ldots, \phi_{i_r}$ and no others, replace I_1 in each of the r occurrences by I_q. The result is still a solution by (*b*), since I_q satisfies Condition 2. (The proof is similar to the proof for Condition 4 in Theorem 4.3.) The solution is still minimal by (*a*). We have shown then that in a minimal solution an FF which does not satisfy Condition 5 can be replaced by an FF which does satisfy Condition 5. It follows from these considerations (and from the obvious fact that there exists a minimal solution) that there exists a minimal solution in which all of the FF's satsify Condition 5.

Theorem 4.5. There exists a minimal solution composed exclusively of prime implicants; more precisely, there exists a minimal solution in which every conjunction of literals which appears in the solution is an r^{th} order prime implicant for the outputs in whose formulas it appears, r being the number of all such outputs.

The proof of this theorem is immediate from the preceding theorems.

5. HEURISTIC REMARKS: COVERAGE

In this section we shall offer some heuristic considerations which will aid the understanding not only of Section 4 but also of the general problem of synthesizing by means of normal formulas. We say that an FF *covers* a row in the truth table (or input combination) if it has the value 1 in that row. Then, if we consider any of the given output functions ϕ_i, the problem becomes one of selecting a set of FF such that every row in which ϕ_i is specified to be 1 is covered by some FF in the set and such that no row in which ϕ_i is specified to be 0 is covered by any FF in the set. Then a solution for ϕ_i is a disjunction of the FF in the set. It is easy to see that this formula will be in state 1 when ϕ_i is specified to be in state 1 and in state 0 when ϕ_i is specified to be in state 0. Of course, we wish a minimal solution not only for ϕ_i but for all the output functions at once. It is interesting to see, in terms of the concept of coverage, why it suffices to consider only prime implicants as indicated by Theorem 4.5. We shall not give another proof of Theorem 4.5, but only those considerations we think are interesting.

Let us go through the five conditions in the definition of an r^{th} order prime implicant I for the output functions $\phi_{i_1}, \ldots, \phi_{i_r}$, in order to see their appropriateness. Condition 1 states simply that I is an FF. Here we note only that a conjunction of literals, one of which is the negation of one of the others, has no coverage at all.

Condition 2 states that I may not cover any of the rows in which $\phi_{i_1}, \ldots, \phi_{i_r}$ is specified to be 0.

Condition 3 states that I must cover at least one row in which $\phi_{i_1}, \ldots, \phi_{i_r}$ are all 1. It is easy to see that if for any output ϕ_{i_j}, if I does not cover at least one row in which ϕ_{i_j} is specified to be 1, it is useless as far as ϕ_{i_j} is concerned. Such a row for ϕ_{i_j} must be a specified case for the other outputs as well, in view of our stipulation that the specified cases are the same for all outputs. Since I may not cover any rows in which any of $\phi_{i_1}, \ldots, \phi_{i_r}$ is specified to be 0, it follows that I must cover a row in which all of $\phi_{i_1}, \ldots, \phi_{i_r}$ are specified to be 1, which is what Condition 2 states.

Let I and I' be two FF each of which satisfies Conditions 1, 2 and 3. Then any specified case which either I or I' covers must be a row in which $\phi_{i_1}, \ldots, \phi_{i_r}$ are all specified to be 1. We say that the coverage of I' is *better than* the coverage of I if I' covers all the specified cases which I covers and at least one specified case which I does not cover. They have *equal coverage* if they cover exactly the same specified cases. Note that there may exist pairs of FF satisfying Conditions 1, 2 and 3 of unequal coverage of which neither is better than the other. This happens when I' covers a specified case which I does not cover and likewise I covers a specified case which I' does not cover.

In these terms, Condition 4 states that there does not exist an I' satisfying Conditions 1, 2 and 3 of equal or better coverage with fewer

literals. For if there were such an I', I' could replace I resulting in a rectifier net of less cost.

Condition 5 states that there does not exist an I' satisfying Conditions 1, 2 and 3 with the same number of literals and with better coverage. For if there were such an I', I' could replace I resulting in a rectifier net having the same cost. Note that Condition 5 is included in the definition of r^{th} order prime implicant in order to shorten the list of prime implicants as much as possible. If we were interested in obtaining all possible minimal solutions (which we are not), we would have to withdraw Condition 5.

Indeed, it may be that a number of r^{th} order prime implicants for $\phi_{i_1}, \ldots, \phi_{i_r}$ all have exactly the same number of literals and all have equal coverage. It would then be expedient to dismiss all of these but one, since they are all of the same value (their coverage) and since they all have the same cost (the number of literals). The choice of which one to retain is not difficult; any one might be chosen at random; but if we wish to be systematic, we may arrange all the FF in some lexicographic order and take the first.

We close this section with a brief discussion of the problem of synthesizing rectifier nets in which the specified cases for the various outputs are not necessarily the same. In what seems to be an appropriate definition of *prime implicant* for this problem, Conditions 1 and 2 are the same as those given in Section 4. Condition 3 must be changed to read as follows:

$$I \,\&\, \psi_{T^{i_1}},\ I \,\&\, \psi_{T^{i_2}},\ \ldots,\ I \,\&\, I_{T^{i_r}} \text{ are each consistent.} \qquad (3')$$

Condition 4(b) must be changed to

$$\psi_{T^{i_1}} \rightarrow (I \rightarrow I'),\ \ldots,\ \psi_{T^{i_r}} \rightarrow (I \rightarrow I') \text{ are each valid.} \qquad (4b')$$

Condition 5(b) must be changed to ($5b'$) which is the same as ($4b'$). Condition 5(c) must be changed to

$$\psi_{T^{i_1}} \rightarrow (I \equiv I'),\ \ldots,\ \psi_{T^{i_r}} \rightarrow (I \equiv I') \text{ are } \textit{not} \text{ all valid.} \qquad (5c')$$

We note (without proof) that, for the narrower problem of this paper, these revised conditions taken together are equivalent to the original conditions taken together. The original conditions were presented (and these revised conditions suppressed, in spite of their greater generality) for greater ease in the search for prime implicants. In particular, Condition 3′ is much more difficult to apply than Condition 3.

The heuristic discussion for this problem is almost the same as the discussion of the early part of this section except that the rows to be covered by an r^{th} order prime implicant are the rows in which none of the $\phi_{i_1}, \ldots, \phi_{i_r}$ are specified to be 0 and at least one is specified to be 1.

The definitions of "better coverage" and "equal coverage" are in terms of this class of cases. Because we are of the opinion that this problem comes up infrequently, we shall not discuss it further. Muller's method (in the paper referred to in footnote 3 of this paper) does not seem to accommodate itself to this problem.

6. A SAMPLE PROBLEM

To find a minimal solution for an m-output rectifier net problem we divide the work into two stages. The first stage is writing, for each r, $1 \leq r \leq m$, and for each set of r outputs $\phi_{i_1}, \ldots, \phi_{i_r}$, the set of the

TABLE III.—*Truth Table.*

	p	q	r	s	t	ϕ_1	ϕ_2	ϕ_3	ϕ_4
31	1	1	1	1	1				
30	1	1	1	1	0				
29	1	1	1	0	1				
28	1	1	1	0	0	0	0	0	0
27	1	1	0	1	1				
26	1	1	0	1	0	0	0	1	1
25	1	1	0	0	1	0	1	1	1
24	1	1	0	0	0				
23	1	0	1	1	1				
22	1	0	1	1	0	0	0	0	1
21	1	0	1	0	1	0	0	1	0
20	1	0	1	0	0				
19	1	0	0	1	1	1	1	0	0
18	1	0	0	1	0				
17	1	0	0	0	1				
16	1	0	0	0	0				
15	0	1	1	1	1				
14	0	1	1	1	0	1	1	0	1
13	0	1	1	0	1	1	1	1	0
12	0	1	1	0	0				
11	0	1	0	1	1	1	0	0	0
10	0	1	0	1	0				
9	0	1	0	0	1				
8	0	1	0	0	0				
7	0	0	1	1	1	1	1	1	1
6	0	0	1	1	0				
5	0	0	1	0	1				
4	0	0	1	0	0				
3	0	0	0	1	1				
2	0	0	0	1	0				
1	0	0	0	0	1				
0	0	0	0	0	0				

r^{th} order prime implicants for these outputs. The second stage is selecting, from these $2^m - 1$ sets, FF which will constitute a minimal solution. We have no theoretically satisfactory procedure for the second stage, but we have a few suggestions which will be presented at appropriate points in our discussion of the sample problem in this section. We also have some suggestions as to a good procedure for the first stage; these suggestions have been left for this section because so far we have not found any interesting formal significance for them.

Note that $2^m - 1$ is a large number of sets of prime implicants, for example, over one thousand, if $m \geq 10$. However, there is a possibility that in such a problem many of these sets will be repeated; for, as noted in Section 4, a set is uniquely determined given the set of input combinations for which all the outputs concerned are specified to be 1. For example, if $\psi_T{}^1$ & $\psi_T{}^2$ & $\psi_T{}^5$ is logically equivalent to $\psi_T{}^1$ & $\psi_T{}^2$ & $\psi_T{}^3$ & $\psi_T{}^4$, then the third order prime implicants for ϕ_1, ϕ_3 and ϕ_5 will be exactly the same as the fourth order prime implicants for ϕ_1, ϕ_2, ϕ_3 and ϕ_4. Moreover, when m is large, the chances are good

TABLE IV.—*Tables for first order prime implicants.*

$\psi_T{}^1$

	p	q	r	s	t
7	0	0	1	1	1
11	0	1	0	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1

$\psi_F{}^1$

	p	q	r	s	t
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_T{}^2$

	p	q	r	s	t
7	0	0	1	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1
25	1	1	0	0	1

$\psi_F{}^2$

	p	q	r	s	t
11	0	1	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_T{}^3$

	p	q	r	s	t
7	0	0	1	1	1
13	0	1	1	0	1
21	1	0	1	0	1
25	1	1	0	0	1
26	1	1	0	1	0

$\psi_F{}^3$

	p	q	r	s	t
11	0	1	0	1	1
14	0	1	1	1	0
19	1	0	0	1	1
22	1	0	1	1	0
28	1	1	1	0	0

$\psi_T{}^4$

	p	q	r	s	t
7	0	0	1	1	1
14	0	1	1	1	0
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0

$\psi_F{}^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
19	1	0	0	1	1
21	1	0	1	0	1
28	1	1	1	0	0

that many of the sets of prime implicants will be empty. Thus, if $\psi_T{}^1$ & $\psi_T{}^2$ & $\psi_T{}^4$ & $\psi_T{}^5$ & $\psi_T{}^6$ is inconsistent, then there are no fifth order (or higher order) prime implicants for ϕ_1, ϕ_2, ϕ_4, ϕ_5 and ϕ_6.

The problem, one of whose minimal solutions we shall find in this section, is given in Table III, an incomplete truth table. In that table it may be observed that the specified cases are those in which exactly three input letters are 1. This fact will make possible a simplifying assumption that will be used in the search for prime implicants.

Tables IV through VIII are equivalent in use to the formulas $\psi_T{}^1$,

TABLE V.—*Tables for second order prime implicants.*

$\psi_T^1 \,\&\, \psi_T^2$

	p	q	r	s	t
7	0	0	1	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1

$\psi_T^1 \,\&\, \psi_T^3$

	p	q	r	s	t
7	0	0	1	1	1
13	0	1	1	0	1

$\psi_T^1 \,\&\, \psi_T^4$

	p	q	r	s	t
7	0	0	1	1	1
14	0	1	1	1	0

$\psi_F^1 \vee \psi_F^2$

	p	q	r	s	t
11	0	1	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_F^1 \vee \psi_F^3$

	p	q	r	s	t
11	0	1	0	1	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_F^1 \vee \psi_F^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_T^2 \,\&\, \psi_T^3$

	p	q	r	s	t
7	0	0	1	1	1

$\psi_T^2 \,\&\, \psi_T^4$

	p	q	r	s	t
7	0	0	1	1	1
14	0	1	1	1	0

$\psi_T^3 \,\&\, \psi_T^4$

	p	q	r	s	t
13	0	1	1	0	1

$\psi_T^1, \psi_T^2, \psi_T^3, \psi_T^4, \psi_F^1, \psi_F^2, \psi_F^3, \psi_F^4$ discussed in Section 4. Instead of forming these as developed disjunctive normal formulas, we use tabular forms which seem easier to work with. It may be observed that rows in these tables correspond to the disjuncts of developed disjunctive normal form. For the sake of future cross-reference the decimal number equivalent to the binary numbers in the rows have been written in the column to the left of each table.

As an aid in finding the second order prime implicant, we form Tables V and VI corresponding to $\psi_T^1 \,\&\, \psi_T^2$, $\psi_T^1 \,\&\, \psi_T^3$, $\psi_T^1 \,\&\, \psi_T^4$, $\psi_T^2 \,\&\, \psi_T^3$, $\psi_T^2 \,\&\, \psi_T^4$, $\psi_T^3 \,\&\, \psi_T^4$ and $\psi_F^1 \vee \psi_F^2$, $\psi_F^1 \vee \psi_F^3$, $\psi_F^1 \vee \psi_F^4$, $\psi_F^2 \vee \psi_F^3$, $\psi_F^2 \vee \psi_F^4$, $\psi_F^3 \vee \psi_F^4$. Tables VII and VIII are for the third and fourth order prime implicants. The usefulness of these tables lies in the fact that with them we can see by inspection whether a given FF satisfies Conditions 2 and 3 in the definition of an r^{th} prime implicant for $\phi_{i_1}, \ldots, \phi_{i_r}$ $(1 \leq r \leq m)$. For example, $pq\bar{r}$ satisfies Condition 2 if and only if in the table for $\psi_F^{i_1} \vee \ldots \vee \psi_F^{i_r}$ there is no row having 1 in the p column, 1 in the q column and 0 in the r column; and $pq\bar{r}$ satisfies Condition 3 if there is at least one row in the table for $\psi_T^{i_1} \,\&\, \ldots \,\&\, \psi_T^{i_r}$ having 1's in the p and q columns and 0 in the r column.

We now find all of the first order prime implicants for the output function ϕ_1. The task is one of sifting through all of the $3^n - 1$ FF and finding which ones satisfy Conditions 1 through 5. (That there are $3^n - 1$ FF follows from the fact that each letter either appears negated, appears unnegated, or does not appear, in any FF.) Because of Conditions 4 and 5, it is expedient to consider those FF with fewer literals before those with many literals. Starting with one literal FF we consider first those satisfying Condition 2. Since, in Table IV, the table labeled $\psi_F{}^1$, column p does not have any 0, we see that $\bar{p}$ satisfies Condition 2 (that is, $\bar{p}$ & $\psi_F{}^1$ is inconsistent). In the same Table IV, since table $\psi_T{}^1$ does have a zero in the column for p we see that $\bar{p}$ also satisfies Condition 3 (that is, $\bar{p}$ & $\psi_T{}^1$ is consistent). Condition 4 is satisfied since any I' would have to be an FF of less than one literal to satisfy Condition 4(a). Since $\bar{p}$ is the only one literal FF satisfying Condition 2, Condition 5 is also satisfied by $\bar{p}$ since any I' satisfying Condition 5(a) will fail to satisfy Condition 2. Hence, we conclude that $\bar{p}$ is a first order prime implicant for ϕ_1. Moreover, it is the only such with one literal.

TABLE VI.—*Tables for second order prime implicants.*

$\psi_T{}^2 \,\&\, \psi_T{}^3$

	p	q	r	s	t
7	0	0	1	1	1
13	0	1	1	0	1
25	1	1	0	0	1

$\psi_F{}^2 \vee \psi_F{}^3$

	p	q	r	s	t
11	0	1	0	1	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_T{}^2 \,\&\, \psi_T{}^4$

	p	q	r	s	t
7	0	0	1	1	1
14	0	1	1	1	0
25	1	1	0	0	1

$\psi_F{}^2 \vee \psi_F{}^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_T{}^3 \,\&\, \psi_T{}^4$

	p	q	r	s	t
7	0	0	1	1	1
25	1	1	0	0	1
26	1	1	0	1	0

$\psi_F{}^3 \vee \psi_F{}^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
28	1	1	1	0	0

TABLE VII.—*Tables for third order prime implicants.*

$\psi_T^1 \,\&\, \psi_T^2 \,\&\, \psi_T^3$

	p	q	r	s	t
7	0	0	1	1	1
13	0	1	1	0	1

$\psi_T^1 \,\&\, \psi_T^2 \,\&\, \psi_T^4$

	p	q	r	s	t
7	0	0	1	1	1
14	0	1	1	1	0

$\psi_F^1 \vee \psi_F^2 \vee \psi_F^3$

	p	q	r	s	t
11	0	1	0	1	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_F^1 \vee \psi_F^2 \vee \psi_F^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_T^1 \,\&\, \psi_T^3 \,\&\, \psi_T^4$

	p	q	r	s	t
7	0	0	1	1	1

$\psi_T^2 \,\&\, \psi_T^3 \,\&\, \psi_T^4$

	p	q	r	s	t
7	0	0	1	1	1
25	1	1	0	0	1

$\psi_F^1 \vee \psi_F^3 \vee \psi_F^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

$\psi_F^2 \vee \psi_F^3 \vee \psi_F^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
26	1	1	0	1	0
28	1	1	1	0	0

TABLE VIII.—*Table for fourth order prime implicants.*

$\psi_T^1 \,\&\, \psi_T^2 \,\&\, \psi_T^3 \,\&\, \psi_T^4$

	p	q	r	s	t
7	0	0	1	1	1

$\psi_F^1 \vee \psi_F^2 \vee \psi_F^3 \vee \psi_F^4$

	p	q	r	s	t
11	0	1	0	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1
26	1	1	0	1	0
28	1	1	1	0	0

Before proceeding, it is well to notice that whenever an FF is subsumed by a prime implicant already in the list (that is, all the literals of the latter are in the former), it need not be considered since it cannot then satisfy Condition 4. Unfortunately, subsumption is not the only way in which an FF is excluded by means of Condition 4. But a subsumption is very quickly noticed and time can thereby be saved.

We continue our search for first order prime implicants for ϕ_1 by considering two literal FF. Since $\bar{p}$ is already known to be in the list we need not consider any FF which subsumes it; in other words, any which contains $\bar{p}$. Starting with p and q columns in the table for $\psi_F{}^1$ (of Table IV), we see whether there is a row with 10 (that is, a row with 1 under p and 0 under q), and then see if there is a row with 11. (Were it not for $\bar{p}$ being already in the list we would also look for 00 and 01; we would have found, for example, that $\bar{p}\bar{q}$ satisfies Condition 2 since there is no 00 in the table for $\psi_F{}^1$, and satisfies Condition 3 since there is a 00 in $\psi_T{}^1$. But, of course, since $\bar{p}\bar{q}$ subsumes $\bar{p}$ it does not satisfy Condition 4.) Since there are rows with 10, $p\bar{q}$ does not satisfy Condition 2, and since there are no rows with 11, pq does not satisfy Condition 2. An examination in like manner of the p and r, p and s, and p and t columns shows that there are no two literal FF in the list containing these pairs of letters.

We continue systematically to sift through the FF with two literals by considering the columns under q and r in the table $\psi_F{}^1$, still in Table IV. We find that $\bar{q}\bar{r}$ satisfies Conditions 2 and 3. Since $\bar{p}$ is the only first order prime implicant for ϕ_1 with one literal and since $\psi_T{}^1 \rightarrow (\bar{q}\bar{r} \rightarrow \bar{p})$ is not valid, Condition 4 is satisfied by $\bar{q}\bar{r}$. (We note that the invalidity of $\psi_T{}^1 \rightarrow (\bar{q}\bar{r} \rightarrow \bar{p})$, or, what is the same, the consistency of $\psi_T{}^1 \,\&\, \bar{q}\bar{r} \,\&\, p$, can be verified by the observation in the table for $\psi_T{}^1$ that a row exists in which p is 1, q is 0 and r is 0, namely, row 19. In this manner any of the tests required by Conditions 4 or 5 can be made by inspecting an appropriate table.) We suspend judgment on whether or not $\bar{q}\bar{r}$ satisfies Condition 5 until all other FF of two literals satisfying Conditions 1 through 4 have been found. The next and last FF with two literals satisfying Conditions 1 through 4 is *st*. We are now in a position to test $\bar{q}\bar{r}$ under Condition 5 and we find that it fails because of *st*. *st* of course passes. All other FF of more than two literals will fail to pass Conditions 2, 3 or 4, and so we have completed the list of first order prime implicants for ϕ_1.

We now search for second order prime implicants for ϕ_1. There seems to be no need to present the details of the remainder of the search for prime implicants; the results have been listed in Tables IX and X. There is one table for each output. Starting at the top of the tables to the left are listed the higher order prime implicants. For example, $\bar{p}\bar{q}$ is listed at the very top of each table, since it is a fourth order prime implicant for each output. As we continue down the list

TABLE IX.—*Tables for prime implicants.*

the order of the prime implicants decreases until we come to those of first order at the bottom. The number to the left shows which of the output functions a set of prime implicants serves. For example, in the table for ϕ_1, $\bar{p}\bar{q}$, $\bar{p}\bar{s}$ and $\bar{p}rt$ are third order prime implicants for ϕ_1, ϕ_2 and ϕ_3. The numbers at the top of the table for an output are the numbers of the rows in Table III which are specified as 1 for that output. Thus, the numbers 7, 11, 13, 14 and 19 appear at the top

TABLE X.—*Excised tables for prime implicants.*

ϕ_1

		7	11	13	14	19
1,2,3	$\bar{p}rt$	X		X		
1,2,4	$\bar{p}rs$	X			X	
1,2	$\bar{p}\bar{r}$	X		X	X	
	$\bar{q}\bar{r}$					X
	qst	X				X
1,3	$\bar{p}\bar{r}t$	X		X		
1,4	$\bar{p}rs$	X			X	
1	$\bar{p}$	X	X	X	X	
	st	X	X			X

ϕ_2

		7	13	14	19	25
1,2,3	$\bar{p}rt$	X	X			
1,2,4	$\bar{p}rs$	X		X		
2,3,4	$\bar{p}\bar{q}\bar{r}s$	X				X
1,2	$\bar{p}\bar{r}$	X	X	X		
	$\bar{q}\bar{r}$				X	
	qst	X			X	
2,3	$\bar{p}\bar{q}\bar{r}s$	X				
	qst		X			X
	$\bar{p}rt$	X	X			X
2,4	$\bar{p}\bar{q}\bar{s}$	X				
	$\bar{p}rs$	X		X		X
2	$\bar{p}\bar{q}\bar{s}$	X	X	X		
	$\bar{p}\bar{q}\bar{r}$				X	
	$\bar{p}\bar{r}\bar{s}$					X
	$\bar{p}rt$				X	X
	qst	X			X	
	$q\bar{s}t$		X			X

ϕ_3

		7	13	21	25	26
1,2,3	$\bar{p}rt$	X	X			
2,3,4	$\bar{r}\bar{s}$				X	
1,3	$\bar{p}\bar{r}t$	X	X			
2,3	pq	X				
	$\bar{r}s$				X	
	$q\bar{s}t$		X		X	
	$\bar{p}\bar{r}t$	X	X			
3,4	$\bar{p}\bar{q}$	X				
	$\bar{r}\bar{s}$				X	
	$\bar{r}t$					X
	$pq\bar{r}$				X	X
3	$\bar{r}t$	X	X	X		
	$\bar{s}t$		X	X	X	
	$\bar{p}t$					X
	$pq\bar{r}$				X	X

ϕ_4

		7	14	22	25	26
1,2,4	$\bar{p}rs$	X	X			
2,3,4	$\bar{p}\bar{q}$	X				
	$\bar{r}\bar{s}$				X	
1,4	$\bar{p}rs$	X	X			
2,4	$\bar{p}\bar{q}$	X				
	$\bar{r}\bar{s}$				X	
	$\bar{p}rs$	X	X			
3,4	$\bar{p}\bar{q}$	X				
	$\bar{r}\bar{s}$				X	
	$\bar{r}t$					X
	$pq\bar{r}$				X	X
4	rs	X	X	X		
	$\bar{s}t$		X	X		
	$\bar{r}s$				X	
	$pq\bar{r}$				X	X

of the table for ϕ_1 since 1's appear in rows 7, 11, 13, 14 and 19 of the table in which ϕ_1 was given. The X's in the table ϕ_i show which rows (which are now columns) are covered by a prime implicant.

This completes the first stage of the problem. We now proceed to the second stage. With the end in mind of making easier the selection of appropriate prime implicants we next consider heuristic methods for excising from the lists unnecessary prime implicants, that is, prime implicants which are *obviously* dispensable in the sense that minimal solutions exist without them. We start with the table for ϕ_1, observing

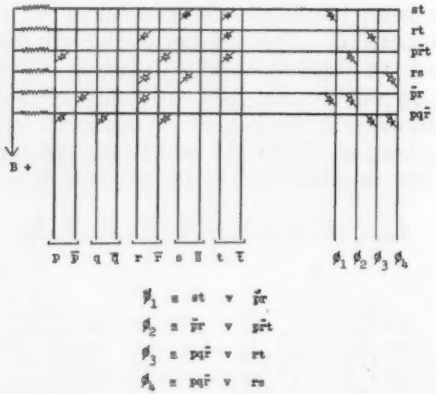


FIG. 7. A minimal solution.

that there are only two X's in the column labeled 11. From this it may be seen that either $\bar{p}$ or st must appear in the solution for ϕ_1. But column 7 is covered both by $\bar{p}$ and by st. Hence, we may drop all prime implicants which cover only column 7 (that is, we may drop all occurrences of $\bar{p}\bar{q}$). The result of a number of such deletions is shown in the excised tables for prime implicants. From these tables we have found a minimal solution, whose cost is 22 (shown in Fig. 7). We omit the proof that it is minimal.

MEDAL DAY PROCEEDINGS AT THE FRANKLIN INSTITUTE

October 16, 1957

RECEPTION AND DINNER

On Wednesday, October 16, 1957, approximately 365 Institute members and guests gathered to pay tribute to the men whose achievements had been recognized by the Committee on Science and the Arts during the past year. A reception honoring the 1957 Medalists was held at 6:30 P.M. in the lobby of Franklin Hall, and at 7:10 the guests assembled for dinner in the Hall. The Very Reverend J. Joseph Bluett, President of St. Joseph's College, asked the blessing.

During dinner, Dr. S. Wyman Rolph, President of the Institute, thanked the members of the Hostess Committee for their assistance at the reception and for the floral decorations. Dr. Rolph proposed a toast to Benjamin Franklin.

Mr. John H. Neher, Chairman of the Committee on Science and the Arts, introduced the following former Medalists who were present at the dinner.

FORMER MEDALISTS ATTENDING MEDAL DAY—1957

Year	Medalist	City
	CRESSON (1848)	
1904	G. H. Clamer	Philadelphia, Penna.
1952	H. Birchard Taylor	Philadelphia, Penna.
	CERTIFICATE OF MERIT (1882)	
1902	Myron Francis Hill	Westport, Connecticut
1921	Joseph S. Hepburn	Philadelphia, Penna.
1939	E. Burke Wilford	Philadelphia, Penna.
1942	Walter Larkin	Philadelphia, Penna.
1944	Western Union Telegraph Company Represented by: H. P. Corwith	New York City
1945	Walter J. Coppock	Moylan, Penna.
	LONGSTRETH (1890)	
1905	Theo. G. Alteneder & Sons Represented by: Theo. G. Alteneder	Philadelphia, Penna.
1911	Joseph S. Hepburn	Philadelphia, Penna.
1922	Joseph F. Keller	New York City
1923	Harry S. Parks	Pottstown, Penna.
1928	Frank N. Speller	Pittsburgh, Penna.
1940	Games Slayter	Newark, Ohio
1948	Nicholas F. Arone	Upper Darby, Pa.
1955	Richard Y. Case	Philadelphia, Penna.

Year	Medalist	City
	POTTS (1906)	
1948	C. A. Lovell	Summit, New Jersey
	LEVY (1923)	
1938	Stewart S. Kurtz, Jr.	Philadelphia, Penna.
1940	Charles Rosenblum	Princeton, New Jersey
1945	Rupen Eksergian	Philadelphia, Penna.
1948	William H. Cherry	Princeton, N. J.
1951	Albert C. Walker	Short Hills, N. J.
	HENDERSON (1924)	
1937	Rupen Eksergian	Philadelphia, Penna.
	WETHERILL (1925)	
1931	Edward C. Wente	Summit, New Jersey
1954	Clarence N. Hickman	Jackson Heights, N. Y.
	BROWN (1938)	
1955	Charles S. Leopold	Philadelphia, Penna.
	BALLANTINE (1946)	
1952	Walter H. Brattain	Murray Hill, N. J.
1953	David G. C. Luck	Bryn Mawr, Penna.
1956	Kenneth Bullington	Madison, N. J.

Dr. Rolph then paid tribute to the Committee on Science and the Arts, in these words:

"Before proceeding with the formal presentation of the Medals, it seems appropriate that thanks be extended not only to the Sponsors of the 1957 Medalists but to all of the members of the Committee on Science and the Arts who have made these awards possible. These Committee members, numbering seventy engineers and scientists, spend many hours in meetings of the Committee as well as countless hours of their own time in pursuing their conscientious and thorough investigations of each subject brought to their attention. They give freely of their labor in order to maintain the splendid tradition of this unique group which was formed when the Institute was founded 133 years ago, and in order to reward the most competent workers in several fields of human endeavor.

"Each Medalist is sponsored by the Chairman of the Sub-Committee of the Committee on Science and the Arts which recommended him for the award. This Sponsor will present the Medalist and read his citation.

"Ladies and Gentlemen, our warm thanks and appreciation to the gentlemen of the Committee on Science and the Arts are well merited."

STATED MEETING

DR. ROLPH: "Today, October 16, 1957, marks the first Stated Monthly Meeting of The Franklin Institute of the State of Pennsylvania

for this season—and the first since May 15, 1957. The minutes of that last meeting were published in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE. If there are no corrections or additions, I declare them approved as published.

"Now I shall present to you the Executive Vice-President of The Franklin Institute, who will tell us of some of the coming events at our Franklin Institute. Dr. John S. Burlew.

DR. BURLEW: "Dr. Rolph, Ladies and Gentlemen:

"A listing of the public events of The Franklin Institute gives only a partial indication of the breadth of the activities carried out under its auspices. There is not time this evening, however, to review the accomplishments of all the departments of the Institute—including the Laboratories for Research and Development, the Bartol Research Foundation, the Biochemical Research Foundation, the Museum, the Library and the JOURNAL. Thus as you listen to this resume of coming events at the Institute, please remember that there are many other activities going on in these departments that are just as important as those that receive public notice.

"This evening's event is the first of the stated monthly meetings of the Institute. At later ones from now until next June the members of the Institute and their guests will have an opportunity to hear outstanding persons in science and technology speak on a variety of subjects. At the November meeting Dr. Rogers McCullough will discuss the safety of nuclear reactors. In December Colonel Richard Leghorn will reveal a report prepared under the auspices of the National Planning Association dealing with some of the technical aspects of arms control.

"In addition to the stated meetings, there will be the usual series of evening meetings held jointly with various technical societies in the Philadelphia area. At the first of these, a joint meeting with A.I.E.E. on October 30th, Mr. Amos E. Joel of Bell Telephone Laboratories will speak on the application of electronics to telephone switching systems. A week later at a joint meeting with the Institute of Aeronautical Sciences, Dr. Felix Kalinski of Vertol Aircraft Corp., will discuss long range planning by industry.

"The Franklin Institute extends its influence over a wider area through a different type of meeting, that is, a symposium at which several speakers present their views on a technical subject. The first symposium this season will take place next Monday, when six invited speakers will discuss an aspect of air pollution, namely, 'Particulate Emission.' We had a successful symposium on other problems in air pollution last year and we hope to make this an annual event. The speaker at the luncheon that day, which will be open to the public,

will be Mr. Gordon Larson, Executive Vice President of Oxy-Catalyst, Inc. The field of air pollution is one in which The Franklin Institute Laboratories are performing original research that is bringing them recognition from different parts of the country. Another field of research in which the Laboratories have special competence is that of air traffic control. Therefore, we have arranged a second symposium this year that will deal with this subject. It will highlight the proposed improvements in air traffic control that are the goal of the Airways Modernization Board recently appointed by President Eisenhower. General E. R. Quesada, chairman of that board, members of his staff, representatives of industry and of various governmental agencies, will take part in this three-day meeting the middle of December.

"In mentioning these special events, I do not want to overlook a continuing series of events that interest a great many of you—namely, the regular lecture-demonstrations in the Planetarium. This month and next, the Russian and American satellites are the featured topics in the Planetarium shows, including the recently inaugurated 'Sky Tours' on Wednesday and Friday evenings.

"In order to fulfill its obligations as an educational institution, The Franklin Institute frequently arranges special events for school children and teachers. One of these will take place next week, when Prof. Eric Rogers of Princeton University will deliver the Annual Dodge Lecture for Young People. His lecture last year was so well received that we prevailed upon him to come back again. This year his subject will be 'Drops, Splashes and Atoms'—certainly a title that will intrigue a young audience.

"A new educational feature to be started next month is a 'chemistry education day' for teachers. The purpose of this Saturday morning meeting is to acquaint teachers of science in the Philadelphia area with some of the latest developments in chemistry. Later we hope to extend this into a series of such meetings dealing with other science topics.

"Another important way that The Franklin Institute meets its objectives as an educational institution is through the science exhibits in the Museum. In order to keep pace with the rapid progress in science, we must always be planning new exhibits. Thus the Bell Telephone exhibit, after five years of great popularity, was closed last summer in order to bring it up to date. Some of the new displays are still being installed but the exhibit is now open again.

"A new exhibit in the planning stage will cover Basic Chemistry. This will be a magnificent affair that we hope will stimulate boys and girls to become interested in that science. We see it as a very real means of helping industry meet the need for chemists and other trained personnel in the future. Therefore we are approaching the chemical

industry for the financial assistance that is needed to make such an exhibit possible on the scale that we think is necessary to accomplish this purpose.

"There are two other exhibits that are about to be opened to the public. One, furnished by the Leeds & Northrup Company, will deal with atomic energy. It is to be installed next month, and it will have a formal opening on the 19th of November.

"The other one is so nearly ready that we have scheduled a party on Monday the 28th of October for members of the Institute to see one of the world's largest model railroads that is now being installed in the Railroad Room on the ground floor. This exhibit, sponsored jointly by the Pennsylvania Railroad and the Reading Railroad, and with the cooperation of the Eastern Model Railroad Manufacturers' Association, will bring to our visitors an appreciation of some of the factors that affect the operation of a modern railroad. I hope that many of you will take advantage of the special members' night to come to see the new exhibit.

"Thank you."

DR. ROLPH: "Thank you, Dr. Burlew.

"Will the Sponsors please take their places?"

PRESENTATION OF AWARDS

DR. ROLPH: "Ladies and Gentlemen, may I recall for you some words of Benjamin Franklin—words I have used before on such occasions as this. Somehow they seem even more fitting now in the light, or darkness, of the political events of the past year. In the presence of the men to whom we pay tribute tonight, they indeed seem fitting. Dr. Franklin said:

> . . . God grant, that not only the Love of Liberty, but a thorough Knowledge of the Rights of Man, may pervade all the Nations of the Earth, so that a Philosopher [as scientists were called in his day] may set his Foot anywhere on its Surface, and say, "This is my Country."

"A very wise man—Dr. Franklin.

"Let us now proceed with our ceremonies, the awarding of the Medals of The Franklin Institute for 1957.

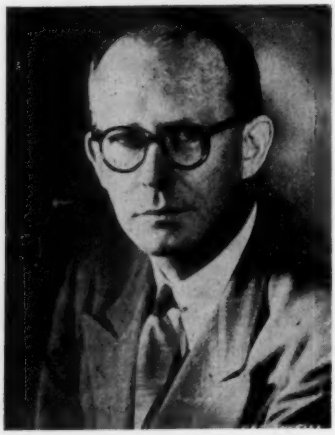
"Each Medalist is sponsored by the Chairman of that Sub-Committee of the Committee on Science and the Arts which recommended him for the award. Each Sponsor will present his Medalist and will read the citation."

PRESENTATION OF THE ELLIOTT CRESSON MEDALS

(Founded in 1848)

This Medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

To Willard Frank Libby, Commissioner, U. S. Atomic Energy Commission, Washington, D. C.; to Reginald James Seymour Pigott, Consulting Engineer, Pittsburgh, Pennsylvania; and to Robert A. Watson-Watt, Radiophysicist, Thornhill, Ontario, Canada.



WILLARD FRANK LIBBY

DR. ROLPH: "The Chair recognizes Dr. Richard M. Sutton."

DR. SUTTON: "Mr. President, I present Willard F. Libby for an award.

"Dr. Libby has helped man to understand the recent millennia of his world by applying knowledge accrued in atomic energy studies of the past decade or two. Seizing upon the discovery of radioactive carbon,—'carbon fourteen,'—which is steadily formed in our atmosphere, by cosmic rays, Dr. Libby has applied the logic and technical skills of the chemist and physicist to disclose the age of animal and vegetable remains containing carbon,—a remarkable tool for archaeology. Now, campfires dead for many centuries and garbage left in caves by prehistoric gormandizers can be dated with considerable accuracy. Up to 25,000 years of man's recent history are thus laid open by the ticking of Geiger counters as they record the explosion of radiocarbon atoms.

"You and I share in the radiocarbon supply of the world. So long as we live, we are in chemical equilibrium with that supply. After death, however, we no longer exchange atoms with the world around us. Thus, six thousand years from now the radiocarbon in our bones, only one-half as active as it is now, would disclose to the atom-wise archaeologist of the year 7957 that certain ancient specimens of *homo sapiens* occupied an almost forgotten city about the year 1957, plus or minus 200 years!

"I present Willard Frank Libby, of Washington, D. C., as a candidate for an Elliott Cresson Medal, 'For his painstaking development of the logic and technique of radiocarbon dating whereby the age of samples of carbon of organic origin may be measured, and for his extensive application of the method to the establishment of a valid world-wide chronological scale.' "

DR. ROLPH: "Dr. Libby, by reason of my office as President of The Franklin Institute of the State of Pennsylvania, I present to you this Elliott Cresson Medal and the Certificate and Report which accompany it."

"On September first of this year, we lost one or our greatly esteemed and faithful members, Mr. Coleman Sellers, 3rd, who was Chairman of the Sub-Committee which investigated the work of R. J. S. Pigott. Tonight the Chair recognizes Mr. W. H. Fulweiler, a member of that Sub-Committee."

MR. FULWEILER: "Mr. President, I present R. J. S. Pigott for an award.

REGINALD J. S. PIGOTT

"Our candidate has had a remarkable engineering career in which he demonstrated his outstanding leadership and ability in a wide

variety of fields including invention, design, development, construction, management, and research.

"Among his most valuable and far-reaching contributions to industry and the engineering profession were the following:

"Studies of the fluid flow in pipes which provided useful design data including formulae for friction of various liquids with different shapes of pipe. These data were more accurate than any previous studies had produced and were later confirmed by other researchers. While the work was developed primarily for the oil field, the results were of general benefit to the hydraulic engineer.

"For the metalworking industry, he produced a radical method of applying coolant to cutting tools resulting in increased tool life and making possible greater production through higher cutting speeds.

"Much of his work was in the field of lubrication and he is largely responsible for the greatly improved lubricating systems in present day automobiles.

"Mr. Pigott holds 36 patents and is the author of approximately 40 technical papers on a variety of subjects.

"He has served as president of five of the national engineering societies and has received many honors for his achievements.

"I present Reginald James Seymour Pigott, of Pittsburgh, Pennsylvania, as a candidate for an Elliott Cresson Medal, 'In consideration of his engineering accomplishments, including his many valuable inventions, his painstaking research, his notable contributions to the technical press, and his engineering leadership.' "

DR. ROLPH: "Mr. Pigott, by the same authority, I present to you this Elliott Cresson Medal and the Certificate and Report which accompany it.

"The Chair recognizes Mr. Alan C. Byers."

MR. BYERS: "Mr. President, I present Robert A. Watson-Watt for an award.

"We shall be forever indebted to the people of Great Britain for their heroic defense of their home islands in the grim days of 1940 and 1941. By efficient use of their pitifully small resources of men and weapons they beat the Luftwaffe to a standstill and cracked the close submarine blockade. That the Battle of Britain was thus won was in large part due to the effective use of radar, in the development of which the foresight and leadership of Sir Robert Watson-Watt played a principal part.

"Early in 1935 Sir Robert suggested a radio location scheme using pulses of electromagnetic energy and shortly carried out an experiment to show that his suggestion had a sound foundation. He then went on to head the initial laboratory group which developed the practical embodiment of this idea. Throughout the war years he continued

to lead the effort which developed the chain of air warning stations, which developed the fighter control tactics, and which led to the identification system and the airborne radar systems.

"Thus, he planted the seed and nurtured the growth of a tree whose fruit influenced all phases of the combat, with most fortunate results.

"I present Robert Alexander Watson-Watt, of Thornhill, Ontario, Canada, as a candidate for an Elliott Cresson Medal, 'For his conception of pulsed radar for the vital air defense of Great Britain, for his solu-

ROBERT A. WATSON-WATT

tions of important technical problems, and for his inspiring leadership which resulted in the timely development of these radar systems.' "

DR. ROLPH: "Sir Robert, by the same authority, I present to you this Elliott Cresson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE EDWARD LONGSTRETH MEDAL
(Founded in 1890)

This Medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes.

To John Bertrand Johnson, Head, Physics Department, Edison Laboratory, Thomas A. Edison Industries, McGraw-Edison Company, West Orange, New Jersey.

DR. ROLPH: "The Chair recognizes Mr. George S. Hartman."

MR. HARTMAN: "Mr. President, I present John B. Johnson for an award.

"The presence of noise in an electric circuit is familiar to everyone; this was particularly true in the early days of radio when static was an almost constant occurrence. Today, any interfering signal of more or less unpredictable nature has become recognized as noise. An interfering wave of a constant frequency is predictable and can be effectively filtered out of any electrical circuit with negligible loss of channel capacity. Thermal noise in resistors is another problem of great concern and Dr. Johnson was a pioneer in the studies conducted in this field. He found that noise was the summation of the effects of very short current pulses of many electrons as they travel between collisions, each pulse having individually a flat spectrum. When such a phenomenon occurs, the resulting noise is determined to be

JOHN BERTRAND JOHNSON

a manifestation of a Brownian movement of electrons passing through the resistor. The experimental work which led to the observation of the phenomenon, its experimental detection and measurement, was indeed a notable step toward a fundamental understanding of the signal-to-noise ratio in all electric circuitry. This work made possible the determination of the lower limit of attainable noise reduction in modern communication receivers, in infra-red spectrometers, and electronic amplifiers. Thus in many respects Dr. Johnson's work forms the foundation for determining the limits set by thermodynamic noise so that scientific investigation need not be continued beyond the limitations imposed by such a phenomenon. It is interesting to note, in passing, that confirmation of Dr. Johnson's work was obtained by subsequent theoretical investigations of Nyquist and others who used statistical studies and quantum mechanical expressions to obtain the same results.

"I present John Bertrand Johnson, of West Orange, New Jersey, as a candidate for an Edward Longstreth Medal, 'In consideration of his fundamental investigations leading to the recognition, measurement, and understanding of the thermal noise in resistors which forms the basis for the understanding of signal-to-noise ratio in electrical circuits and justly bears the name "Johnson-Noise".' "

DR. ROLPH: "Dr. Johnson, by the same authority, I present to you this Edward Longstreth Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE LOUIS E. LEVY MEDAL
(Founded in 1923)

This Medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

To David Barnard Steinman, Consulting Engineer, New York, New York.

DR. ROLPH: "The Chair recognizes Dr. Joseph S. Hepburn."

DR. HEPBURN: "Mr. President, I present David Barnard Steinman for an award.

DAVID BARNARD STEINMAN

"In his excellent paper on the design of the Mackinac Bridge for aerodynamic stability, Dr. Steinman has made a concise and comprehensive presentation of the engineering principles involved in his design of this suspension bridge.

"The outstanding original feature contributing to aerodynamic

stability is the provision of wide open spaces between the stiffening trusses and the outer edges of the roadway. An additional factor for such stability is longitudinal open spaces in the bridge floor between the roadways.

"The contribution of these features to such stability was confirmed by wind tunnel tests on scale models. Previous suspension bridges have had critical wind velocities ranging from 30 to 80 miles per hour. The Mackinac Bridge as designed has no critical wind velocity.

"I present David Barnard Steinman, of New York, as a candidate for the Louis E. Levy Medal, 'In recognition of his outstanding paper, "The Design of the Mackinac Bridge for Aerodynamic Stability," appearing in the December, 1956 issue of the JOURNAL OF THE FRANKLIN INSTITUTE.' "

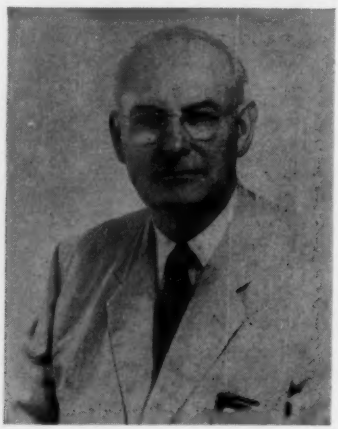
DR. ROLPH: "Dr. Steinman, by the same authority, I present to you this Louis E. Levy Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE GEORGE R. HENDERSON MEDAL
(Founded in 1924)

This Medal is awarded for meritorious inventions or discoveries in the field of Railway Engineering.

To the Association of American Railroads.

DR. ROLPH: "The Chair recognizes Mr. H. L. Decker."



WILLIAM T. FARICY

MR. DECKER: "Mr. President, I present William T. Faricy, Chairman of the Board of the Association of American Railroads, for an award to that Association.

"The Association, through its vast program of research and develop-

ment, covering such phases as safety, standardization of materials, establishment of specifications for freight and passenger equipment and methods of reducing maintenance and increasing efficiency, has produced many outstanding developments such as the tightlock coupler, the standard passenger and freight car axle, the solid journal bearing, higher capacity draft gears, high speed freight trucks, improved rail designs, establishment of maximum wheel and rail loads, and standardization of equipment. These are but a few of the Association's many achievements which have contributed to making American railroads a mass transportation system unexcelled from the standpoint of efficiency, economy and safety.

"I present William T. Faricy, of the Association of American Railroads, to receive, on their behalf, the George R. Henderson Medal, 'In recognition of the many achievements of the Mechanical and Engineering Divisions in the many fields of railway engineering such as the standardization of materials and equipment; the development of the tightlock coupler, the standard passenger car axle, and the solid journal bearings.' "

DR. ROLPH: "Mr. Faricy, by the same authority, I present to you on behalf of the Association of American Railroads this George R. Henderson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE JOHN PRICE WETHERILL MEDALS
(Founded in 1925)

This Medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

To Warren W. Carpenter, Research Consultant, Winter Park, Florida; and to The Martin Company, Baltimore, Maryland.

DR. ROLPH: "The Chair recognizes Mr. Walter C. Wagner."

MR. WAGNER: "Mr. President, I present Warren W. Carpenter for an award.

"The area of telephone communication technology in which Mr. Carpenter made his outstanding contributions included a wide range of original developments.

"When Mr. Carpenter joined the Bell System, the automatic switching system was being devised to meet the more complex situations expected in the larger cities. He contributed a basic improvement, the common control unit, in the form of a 'decoder.' This was so eminently effective that it is still a part of our most modern telephone exchanges. He also, in one stroke, doubled the capacity of the customers' dial controls by devising a new form of discriminating arrangement called the 'line finder.'

"One basic invention destined to play a part of the greatest importance was his concept of 'automatic alternate routing,' resulting in the more efficient use of a comparatively smaller number of (transmission) trunks.

"In the field of mechanizing the billing of telephone customers for service rendered, Mr. Carpenter devised processing machinery that records automatically all of the desired billing information. This development culminated in the development of the 'A.M.A.' (Automatic Message Accounting) System which now records and processes billing information on hundreds of millions of telephone messages per year. A.M.A. has eliminated the last major road block to direct distance dialing. Together with superb voice transmission facilities and modern switching facilities, the time is imminent when any tele-

WARREN W. CARPENTER

phone customer will be able to dial any other telephone customer anywhere in the country.

"Among his other important inventions are the new type of switching system incorporating the 'marker' and its 'connector,' which tests and selects available transmission paths; the 'automatic trouble indicator,' to ascertain the condition of related apparatus, and, another, the concept of the 'automatic second trial,' which discharges the controlling mechanism when trouble occurs, and then seizes the same or a new controlling mechanism to complete the desired connection.

"I present Warren W. Carpenter, of Los Angeles, California, as a candidate for a John Price Wetherill Medal, 'For his important contributions in the field of switching, including the invention of the Automatic Message Accounting System and many related inventions fundamental to the technology of modern telephone switching.'"

DR. ROLPH: "Mr. Carpenter, by the same authority, I present to you this John Price Wetherill Medal and the Certificate and Report which accompany it.

"The Chair now recognizes Mr. R. Thomas Sawyer."

MR. SAWYER: "Mr. President, I present G. T. Willey, Vice President-Manufacturing, of The Martin Company for an award to that Company.

"The Martin Company carried out the complete development of a manufacturing process for joining of metal skins to a low density core. This system or sandwich structure has been developed, used, and licensed to other manufacturers by The Martin Company. This accomplishment is not limited to the development of a suitable adhesive

G. T. WILLEY

or bonding agent, but to a manufacturing process for a satisfactory honeycomb core, and a method of joining metal skins to that low-density core by means of the adhesive or bonding to produce an efficient structure. The final success of this process involved the talents of many persons, and has required coordination of the efforts of at least five groups in the company—laboratory, engineering, tooling, manufacturing, and quality control.

"I present G. T. Willey, of The Martin Company, of Baltimore, Maryland, to receive on its behalf a John Price Wetherill Medal, 'In recognition of the intensive research, development and manufacture of bonded structures.' "

DR. ROLPH: "Mr. Willey, by the same authority, I present to you on behalf of The Martin Company this John Price Wetherill Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANK P. BROWN MEDAL
(Founded in 1938)

This Medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

To Pier Luigi Nervi, Architect-Engineer, Rome, Italy.

DR. ROLPH: "The Chair recognizes Mr. Joseph Gray Jackson."

MR. JACKSON: "Mr. President, I present, *in absentia*, Pier Luigi Nervi for an award.

"Professor Nervi, by courage and foresight, has wrought great changes in reenforced concrete. As a result, a wide variety of delicate almost lace-like space frames have appeared. These structures, while sound from an engineering standpoint, are also very appealing to the eye.

PIER LUIGI NERVI

"I present, *in absentia*, Pier Luigi Nervi, of Rome, Italy, as a candidate for the Frank P. Brown Medal, 'In consideration of his outstanding contributions in pioneering new fields in reenforced concrete design, through the use of new processes of construction and new applications of stress analysis, resulting in broadening of architectural and engineering horizons.' "

DR. ROLPH: "On behalf of The Franklin Institute of the State of Pennsylvania, in the absence of Dr. Pier Luigi Nervi, I present to you, Dr. Luciolli, Consul-General of the Republic of Italy, the Frank P. Brown Medal and the Certificate and Report which accompany it. Dr. Luciolli, will you see that they are forwarded to Dr. Nervi?"

PRESENTATION OF THE FRANCIS J. CLAMER MEDAL
(Founded in 1943)

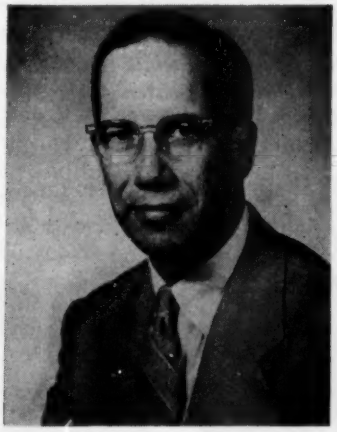
This Medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

To William G. Pfann, Metallurgist, Bell Telephone Laboratories, Murray Hill, New Jersey.

DR. ROLPH: "The Chair recognizes Mr. A. M. Bounds."

MR. BOUNDS: "Mr. President, I present William G. Pfann for an award.

"The recent spectacular growth of the transistor and crystal diode industries is entirely dependent on the availability of low-cost crystals of semi-conducting substances of extremely high purity. Previous to Mr. Pfann's work, a most common method of purification of crystalline solids was that of repeated fractional crystallization. The process was slow, tedious and most unsuitable for industrial production.



WILLIAM G. PFANN

"By continuously passing a molten zone through an impure ingot of metal, the impurities, according to their nature, gather either at the melting face of the zone, or at the recrystallizing or freezing face of the zone. Zone melting was invented to purify germanium for transistors, and in this task its success has been spectacular. The method is not restricted to germanium, however, but can be applied to silicon and other semi-conductors, metals, organic and inorganic compounds. In fact, it is useful for any crystalline substance which can be safely melted and in which there exists an appreciable difference in impurity concentration between the freezing solid and the liquid from which it freezes.

"This invention is important not only because it has unlocked the door to the production of transistors in large volume and at low cost, but also because it has engendered so much research in the purification of metals, and organic and inorganic crystalline substances throughout the scientific world.

"I present William G. Pfann, of Far Hills, New Jersey, as a candidate for the Francis J. Clamer Medal, 'In recognition of his discovery and application of zone refining to metals and other crystalline substances.' "

DR. ROLPH: "Mr. Pfann, by the same authority, I present to you the Francis J. Clamer Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE STUART BALLANTINE MEDALS
(Founded in 1946)

This Medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

To Robert Morris Page, Associate Director of Research for Electronics, U. S. Naval Research Laboratory, Washington, D. C., and Leo Clifford Young, Electronics Consultant, U. S. Naval Research Laboratory, Washington, D. C.

DR. ROLPH: "The Chair recognizes Mr. Lewis P. Tabor."

MR. TABOR: "Mr. President, I present Leo Clifford Young and Robert Morris Page for awards.

ROBERT MORRIS PAGE

LEO CLIFFORD YOUNG

"The year 1933 marks a turning point in the history of military science. It is the year of a technicological break-through that made

possible the development of a powerful method for detecting many significant enemy movements at a distance.

"In 1933, the method of measuring distance by noting the time required for a pulse of electromagnetic energy to travel to a reflecting surface and return was well established. This method had been used with great success since 1925, when it was developed by Doctors Tuve and Breit, for measuring ionosphere heights.

"Also by 1933, it had been definitely established that measurable reflections of electromagnetic energy were returned by moving ships and airplanes. It was recognized, however, that the problems that had to be solved in order to develop a system for locating airplanes in direction and range were many and difficult, and in many areas no well-defined method of attacking these problems was known.

"One of the key problems was the design of a transmitter that would produce very short pulses of very high power at very high frequencies. In this same year, while working on another project, Mr. Leo C. Young discovered the basic method of attack on this problem that led to its very successful solution, and which started the chain of events that led to the development of pulsed radar in the United States. Mr. Young was very soon joined in this work by Dr. Robert M. Page. Mr. Young and Dr. Page, with their small staff, successfully overcame in rapid succession the problems of designing short time-constant receivers, synchronizing and indicating equipment, and all of the other many and difficult circuit and systems problems standing in the way of development of the first successful pulsed radar system.

"I present Leo Clifford Young and Robert Morris Page, both of Washington, D. C., as candidates for Stuart Ballantine Medals, 'In consideration of their outstanding achievement, the development of the first pulsed radar system in the United States.' "

PRESENTATION OF THE FRANKLIN MEDAL
(Founded in 1914)

This Medal is awarded annually to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

To Hugh Stott Taylor, Dean of the Graduate School and the David B. Jones Professor of Chemistry, Princeton University, Princeton, New Jersey.

DR. ROLPH: "The Chair recognizes Dr. H. A. Neville."

DR. NEVILLE: "Mr. President, I have the honor to present to you, as a candidate for the highest award of The Franklin Institute, the distinguished scientist and educator Sir Hugh Taylor, Dean of the Graduate School of Princeton University.

"The scientific contributions of Dr. Taylor have been in the field of physical chemistry, especially in basic studies of the kinetics, mechanism and catalysis of chemical reactions at solid surfaces. In these areas he has pioneered, since World War I, in the experimental investigation of many problems relating to the relative efficiency of catalysts, the effects of catalyst poisons and promoters, the activation energetics of catalyzed reactions. Through his many scientific articles and books he has provided highly original theoretical interpretations of these studies, thereby contributing very significantly to the understanding of catalytic phenomena and to the development of many practical applications of this knowledge.

HUGH SCOTT TAYLOR

"The international scientific reputation of Sir Hugh is attested by the numerous medals and honorary degrees awarded to him in this and in other countries, and by knighthood conferred upon him by Queen Elizabeth II of England and by Pope Pius XII.

"Undoubtedly a major accomplishment of Dr. Taylor has been in his capacity as an educator, in his stimulating guidance and encouragement of many students and co-workers who, in turn, have made and continue to make significant contributions to science. Because of his own scientific investigations and his influence upon other scientists of this generation, he ranks as one of the most distinguished chemists of the world in our time.

"Therefore, Mr. President, with this formal citation: 'In recognition of his many notable contributions to the science of physical chemistry, particularly his development of the widely significant theory of heterogeneous catalysis, and his eminent achievements as author, editor and teacher, 'I present to you, Hugh Stott Taylor, of Princeton, New Jersey, as a candidate for The Franklin Medal."

DR. ROLPH: "Sir Hugh, as President of The Franklin Institute of the State of Pennsylvania, I present to you this highest award of the Institute, The Franklin Medal and the Certificate and Report which accompany it. My warmest congratulations and well wishes, Sir."

ADDRESS BY SIR HUGH STOTT TAYLOR

Following the presentation of the Franklin Medal, Sir Hugh Taylor delivered an informative talk entitled "Deliberate Speed," in which he described his many and fruitful years of research in the field of catalysis. The full text of Sir Hugh's address appears on page 433 of this issue of the JOURNAL.

After Sir Hugh had delivered his talk, Dr. Rolph conveyed to him the thanks of the Institute, dismissed the assembly with a cordial word, and declared the meeting adjourned.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

November 20, 1957

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Dr. S. Wyman Rolph, President, called the meeting to order with approximately 125 members and guests in attendance.

The President stated that the minutes of the Stated Meeting of October 16, 1957, would be printed in the November issue of the JOURNAL and will be submitted for approval at the next Stated Meeting of The Institute.

The President stated that Dr. Hugh Stott Taylor of Princeton University, Franklin Medalist for 1957, had been nominated by the Board of Managers for Honorary Membership to The Institute and this nomination was now being presented to the members for action. Upon motion duly made and seconded, Dr. Taylor was unanimously elected to Honorary Membership.

The President recognized Mr. Morton Gibbons-Neff, Chairman of the Membership Committee, who presented to him the names of two members of The Franklin Institute who had completed 50 years of membership in 1957,—Darthela Clark and Frank Shaw Clark, whose father, Dr. Walton Clark, was president of The Institute for 17 years and a member of The Institute for 45 years. The President presented the gold key and gold membership certificate to Miss Clark, noting with pride that she is the first of her sex to have attained 50 years of membership in The Institute. He then presented the gold key and gold membership certificate to Frank Shaw Clark in absentia, asking Miss Clark to accept the key and certificate for her brother who was unable to be with us on this occasion.

The President then introduced the speaker of the evening, Dr. C. Rogers McCullough of the Monsanto Chemical Co., for this, the third Annual Philip C. Staples Lecture. Dr. McCullough, like Benjamin Franklin, has apportioned part of his life to public service and among other activities is Chairman of the Advisory Committee of the Atomic Energy Commission for Reactor Safeguards and a member of the American Nuclear Society.

Dr. McCullough's subject was "Safe Atoms." He gave an enlightening talk. He spoke of the atomic energy industry as having the second lowest record of accidents in all industry, and explained that there are three ways to protect against radiation: distance, shielding and limiting the time of exposure. Dr. McCullough showed interesting slides on the Pressurized Water Reactor at Shippingport, Pennsylvania.

The talk was exceedingly well received as evidenced by the discussion period following the lecture.

The meeting was adjourned by the President at 9:26 P.M.

WILLIAM F. JACKSON, JR.
Secretary

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 298 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

RECENT ADDITIONS

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MEMBERSHIP

The following elections were approved on November 7, 1957 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Richard M. Armstrong
A. A. Autilio
Roger S. Babcock
Donald A. Bixler
Raoul Blondelle
Edward Brittin
Ervin Bruzel
John G. Bryant
Robert Buhner
Robert Bullick
Robert Bumstead
Harold L. Burmeister, Jr.
Francis H. Buzzard
Barton M. Cahan
C. E. Cain
F. J. Cain, Jr.
Robert Webb Campbell
Malvin Carmin
A. Brinton Carson
Dominick M. Caruso, Jr.
Meyer L. Casman
William K. Cassel
William F. Cessna
C. G. Chandler
A. H. Christian
Robert S. Cook
John Cornell
Louis E. Drummond
Sylvia J. Ellis
George R. Fink
Kurt Wm. Fleischer
Homer P. Geehr
C. I. Geibel, Jr.
R. B. Gerhardt
John R. Grandinetti
Paul V. Guischard
John A. Halbe
Norman Hoffman
Richmond Horrocks
Myer P. Jacobs
William D. Jameson
Arthur J. Julicher
Samuel Kraichman
J. R. Lamplugh
E. K. Lay
Robert A. Martin
Francis J. Mawson
Reuben Miller
George Allen Olsen
Mario Polidori
Cyril S. Rhodes
John C. Rosania
Charles Schaal
Craig A. Schoeller
John E. South
William F. Stahl
Edward C. Stroup
Carl Paul Vogt
John W. Wallenius
John M. Wilson
Felix Zandman

EDUCATOR

Kolman M. Gross
John A. Guischard

NECROLOGY

Margaret Bennett Barringer, '36
J. Kenneth W. Macalpine, '25
Clarence C. Schaefer, '52
Frederick Charles Scholler, '42
Frank L. Thomson, '50
J. Kirby Tompkins, '55
DeForrest P. Willard, '34

THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT
NICOL H. SMITH, DIRECTOR

THE SAMPLED-DATA-ANALOG COMPUTER

BY

GILBERT S. STUBBS[1]

The standard stepping relay is the key computing element of a new "Sampled-Data-Analog Computer" which is being constructed by The Franklin Institute Laboratories' Nuclear Engineering Division. Designed primarily for the solution of partial differential equations, the new computer extends the domain of the analog computer to complex, distributed-parameter problems which were formerly considered the domain of the digital computer.

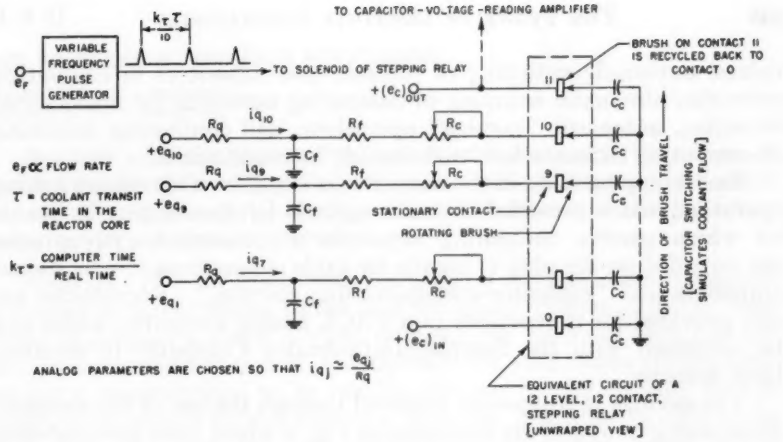
The Sampled-Data-Analog Computer has been designed as a versatile tool for nuclear engineering. Major applications include:

1. Simulation of multi-group, neutron-diffusion equations.
2. Two-dimensional simulation of convective-heat-transfer in
   a. Reactor fuel elements,
   b. Multi-layer shielding of the reactor vessel,
   c. Multi-layer thermal baffles in the reactor and in its cooling system,
   d. Intermediate heat exchangers and steam generators of the reactor power plant;
3. Simulation of the transport delays and mixing effects in a reactor coolant loop;
4. Simulation of natural circulation in the reactor coolant loop, including variations in flow rate caused by temperature transients;
5. Simulation of the formation, decay, and circulation of delayed-neutron precursors in a homogenous reactor loop.
6. Calculation of conversion ratios and burn-up in reactor fuel cycles.

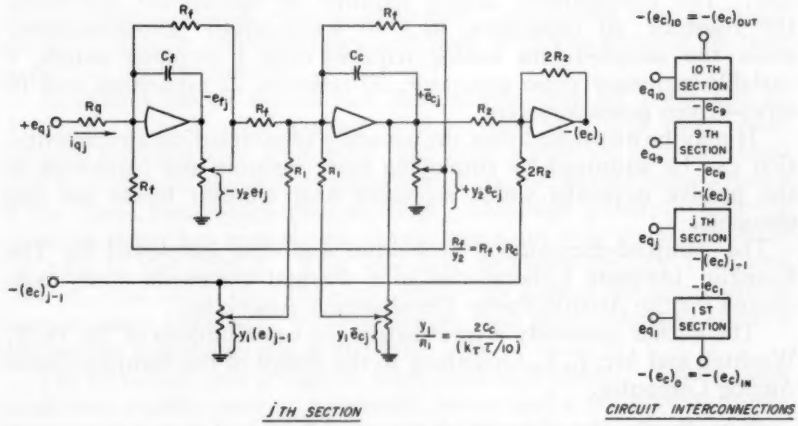
The new computer will find important applications in conjunction with conventional analog equipment, as a tool for studying the dynamics of nuclear reactors and nuclear power plants.

The principal advantage of the Sampled-Data-Analog Computer over conventional analog computers is that its use of stepping relays greatly reduces the number of expensive operational amplifiers, servo-multipliers, and electronic multipliers which are required to represent partial differential equations. Operated by electronic pulse generators, the stepping relay units may perform a variety of operations, including automatic iteration, "read-in" and "read-out" of analog information stored in capacitors, automatic adjustment of resistance and capacitance

[1] Senior Research Engineer, Nuclear Engineering Division.


TO CAPACITOR - VOLTAGE - READING AMPLIFIER
VARIABLE FREQUENCY PULSE GENERATOR
TO SOLENOID OF STEPPING RELAY
BRUSH ON CONTACT 11 IS RECYCLED BACK TO CONTACT 0
DIRECTION OF BRUSH TRAVEL
[CAPACITOR SWITCHING SIMULATES COOLANT FLOW]
STATIONARY CONTACT
ROTATING BRUSH
$e_f \propto$ FLOW RATE
τ = COOLANT TRANSIT TIME IN THE REACTOR CORE
k_τ = COMPUTER TIME / REAL TIME
ANALOG PARAMETERS ARE CHOSEN SO THAT $i_{qj} \simeq \frac{e_{qj}}{R_q}$
EQUIVALENT CIRCUIT OF A 12 LEVEL, 12 CONTACT, STEPPING RELAY [UNWRAPPED VIEW]

Sampled-data analog representation



Conventional analog representation

FIG. 1. Comparison of sampled-data-analog and conventional analog computing methods, in terms of ten-section representations of heat transfer in a reactor core.

Analogous Quantities

Electrical	Thermal
i_{qj}	Space-average heat generation rate in the jth axial section of the reactor core
e_{fj}	Space-average fuel element temperature in the jth section
$(e_c)_{in}$	Space-average coolant temperature at the reactor core inlet
$(e_c)_{out}$	Space-average coolant temperature at the reactor core outlet
C_f	Total heat capacity of fuel elements in the jth section
R_f	Effective heat transfer resistance of fuel elements in the jth section
R_c	Effective heat transfer resistance of coolant film in the jth section
C_c	Total heat capacity of coolant in the jth section

values, automatic switching of resistors and capacitors in computign networks, automatic scanning of computing networks for multi-signal recording, automatic "homing" operations, and continuous switching of computing elements for "time-sharing" computations.

Each stepping relay in the computer is wired for a specific switching operation, and is provided with a receptacle for special plug-in boards on which passive computing networks are mounted. Receptacles are provided on the plug-in boards for cable connections to operational amplifiers and "capacitor-voltage-reading circuits." Receptacles are also provided for connections to a PACE analog computer, which can be combined with the Sampled-Data-Analog Computer to simulate large systems.

The savings in equipment achieved through the use of the sampled-data-analog computer are illustrated in Fig. 1 where both sampled-data and conventional analog methods are compared in terms of a simple ten-section representation of convective heat transfer in a reactor core. The conventional analog requires 30 operational amplifiers, 100 resistors, 20 capacitors, and 40 servo-driven potentiometers,[2] while the sampled-data analog requires only 1 stepping switch, 1 variable-frequency pulse generator, 20 resistors, 22 capacitors, and 10 servo-driven potentiometers.

It may be mentioned that the accuracy of sampled-data representation can be improved by employing more resistors and capacitors in the passive networks which represent heat transfer inside the fuel elements.

The sampled-data-analog technique was first employed by The Franklin Institute Laboratories in a thermal transients study conducted for the Atomic Power Development Associates.

The author gratefully acknowledges the contributions of Dr. G. P. Wachtell and Mr. E. L. Greenberg to the design of the Sampled-Data-Analog Computer.

NOTE: For discussion of another computer especially designed in the Laboratories for nuclear engineering research, see "Reactivity Computer" by Mr. Stubbs in this JOURNAL, Vol. 263, No. 6, June 1957.

[2] Various modifications of this circuit are possible which would require fewer servo-driven potentiometers but more amplifiers. One such modification halves the number of potentiometers but doubles the number of amplifiers.

***Seminar on* Inhibition and Stimulation of Anaerobic Glycolysis of Ascites Tumor Cells.**—DR. W. D. YUSHOK of the Biochemical Research Foundation addressed the seminar of the Foundation on Friday afternoon, May twenty-fourth, 1957.

A review (1) of the many investigations on metabolic and enzymatic activities of cancer tissues shows that malignant tumors have not been found to contain a single enzyme or metabolic pathway that has not been found in normal tissues, and furthermore that tumor tissues have not been demonstrated to be totally deficient in any enzyme that is generally present in normal tissues. However, quantitative differences have been observed between neoplastic and normal tissue which indicate that there is a shift in the balance of metabolic activities during the development of cancer toward a uniform metabolic pattern. Highly dedifferentiated tumors of different histogenesis have relatively similar metabolic characteristics and bear little resemblance to their respective tissues of origin.

Malignant tumor tissues had been shown over 30 years ago by Warburg (2) to convert glucose to lactic acid at a high rate. Subsequent investigations confirmed that high glycolysis is a universal trait of malignant tumor cells regardless of the metabolic characteristics of the tissue from which they develop. This high glycolytic activity of cancer tissue has been shown to take place not only in the test tube but also in the animal. This was first demonstrated by Cori and Cori (3), who found that the blood which has passed through a tumor contains more lactic acid and less glucose than does venous blood in a comparable tumor-free site in the same host.

The glycolytic rate is highest when oxygen is absent. Under these conditions, rapidly growing embryonic tissues and a few normal tissues of mature animals also have an appreciable rate of glycolysis. However, when oxygen is introduced, the production of lactic acid is greatly reduced in both embryonic and adult tissues (2). Cancer cells, on the contrary, persist in producing lactic acid at a high rate even in the presence of oxygen.

Warburg (4) attributed the high aerobic glycolysis of cancer tissue to an irreversibly damaged respiratory system. After many thorough investigations designed to find the site of this suggested damage, Weinhouse (5) concluded that a respiratory defect does not exist in cancer tissue. Weinhouse proposed the following interpretation: ". . . the anaerobic glycolysis of tumor slices is so high that a normal respiration and a normal Pasteur Effect are incapable of eliminating it. This latter statement places the emphasis on the high glycolysis

of tumor slices rather than on the respiration or on the absolute Pasteur Effect, neither of which are quantitatively diminished in neoplastic cells."

One of the main objectives of my investigations here at the Biochemical Research Foundation has been to find an explanation for the high rate of glycolysis in cancer cells. My long range study has been directed toward finding chemical means of controlling the rate of glycolysis of ascites tumor cells. In order to gain a fresh approach to this old but unsolved problem, a preliminary survey was carried out to study the effects of over 100 compounds of many diverse types on anaerobic glycolysis (6). In the course of the work it was shown that while some chemical agents, such as sugar analogs, can effectively suppress glycolysis of ascites tumor cells, other agents of widely different structures are capable of stimulating this already high rate.

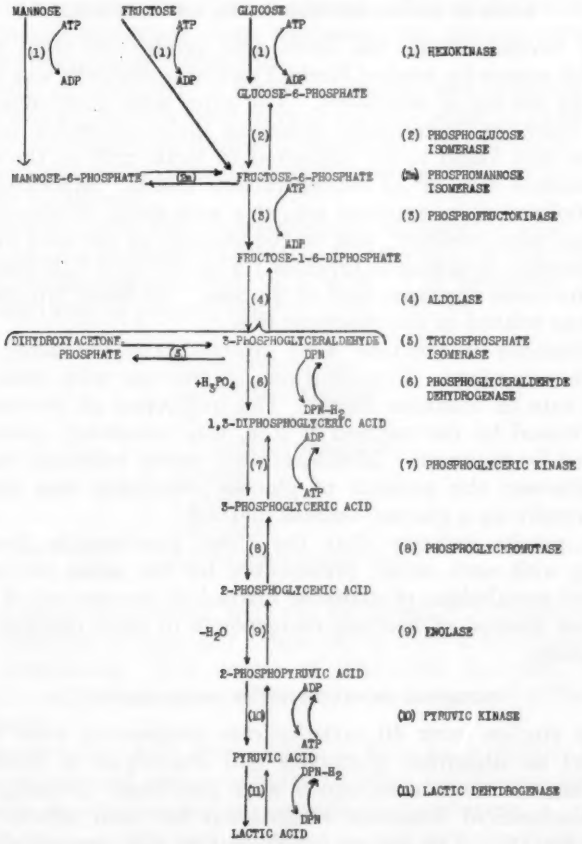
THE EMBDEN-MEYERHOF PATHWAY OF GLYCOLYSIS

The enzymes involved in the anaerobic breakdown of glucose to lactic acid have been thoroughly investigated by Meyerhof, Embden, the Coris and many other workers. The metabolism of glucose and mannose requires the coordinated action of eleven enzymes, which are indicated in their proper sequence in the diagram. The breakdown of fructose involves ten enzymes, since it does not need a phosphohexose isomerase. All three glycolyzable hexoses are phosphorylated by hexokinase (step 1). Glucose phosphorylated at the number 6 carbon is converted to fructose-6-phosphate by phosphoglucose isomerase. Mannose-6-phosphate is not affected by this isomerase but requires a specific phosphomannose isomerase (step 2a). From fructose-6-phosphate to lactic acid, the metabolic pathway is the same for the three hexoses.

The Embden-Meyerhof pathway not only involves the straight chain of enzymes for hexose degradation, but also requires a recycling of complementary coenzymes. Adenosine triphosphate (ATP) is formed in reactions 7 and 10, catalyzed by phosphoglyceric kinase and pyruvic kinase, respectively. It is utilized in reactions 1 and 3 for the phosphorylation of the hexoses and of fructose-6-phosphate. When 1 mole of hexose is converted to lactic acid, 4 moles of ATP are synthesized, while 2 moles are degraded to adenosine diphosphate (ADP), resulting in a net production of 2 moles of ATP. In the living cancer cell, the ATP formed by glycolysis is available as a source of energy for synthesis of cell constitutents of higher molecular weight. The phosphate which is incorporated into proteins, nucleic acids, and lipids can be replenished by the inorganic phosphate which enters the cycle in step 6. To maintain a steady state rate of glycolysis in an *in vitro* system in which the cells are maintained alive but not growing, adenosinetriphosphatase (ATPase) may be involved in breaking down

the excess ATP to ADP and inorganic phosphate, as is suggested by the work of Meyerhof and Wilson (7). Chemical agents which affect ATP metabolism should also alter the rate of glycolysis.

The maintenance of the glycolytic cycle at a steady-state rate is also dependent upon the balanced reduction and oxidation of diphosphopyridine nucleotide (DPN). DPN is reduced in reaction 6 and



The Embden-Meyerhof pathway of glycolysis.

reoxidized in reaction 11. Chemical agents which affect the balance of oxidized and reduced DPN also should change the rate of glycolysis.

The glycolytic process not only yields ATP as a source of energy for synthesis but it also furnishes 3-carbon compounds which can be used as "building blocks" for the synthesis of substances of higher molecular weight. From the point of view of thermodynamics, glycoly-

sis is considered to be inefficient in its yield of energy when compared to the complete oxidation of glucose. On the other hand, from the standpoint of synthesis of protoplasm, glycolysis should not be regarded as inefficient, because of the intermediate metabolites which it makes available. To resynthesize these intermediates from carbon dioxide and water requires considerable energy.

RATES OF GLUCOLYSIS, FRUCTOLYSIS, AND MANNOLYSIS

In my investigations, the lactic acid production from the three glycolyzable sugars by washed Krebs 2 ascites tumor cells was measured prior to the testing of inhibitors. The rates were determined by the standard Warburg manometric technique under anaerobic conditions.

Glucose was found to be converted to lactic acid at the same apparent maximal rate at all concentrations tested. Mannose likewise was metabolized at its maximal rate at a wide range of concentrations. Its maximal rate, however, was on the average 28 per cent lower than that of glucose. Fructose at levels of 1.5×10^{-2} to 0.1 M was metabolized at the same apparent rate as glucose. At lower fructose levels, the rate was related to the concentration.

Combinations of fructose with glucose gave the same maximal rate as glucose alone. Combinations of fructose with mannose had the same rate as mannose alone. The utilization of fructose, which was determined by the method of Roe, was completely prevented by glucose and by mannose. Mannose, also, partly inhibited the utilization of glucose; the amount of glucose remaining was determined colorimetrically by a glucose oxidase method.

These results indicate that the three glycolyzable hexoses are competing with each other, presumably for the same enzyme. The preferential metabolism of mannose instead of glucose and of mannose and glucose instead of fructose corresponds to their relative affinities to hexokinase.

INHIBITION OF GLYCOLYSIS BY SUGAR ANALOGS

In my studies, over 40 carbohydrate compounds were tested for their effect on anaerobic glucolysis and fructolysis of Krebs ascites cells. Many of these compounds were previously investigated here at the Biochemical Research Foundation for their effects on yeast fermentation (8). The sugars considered in this presentation are all in the D-series.

The only sugar analogs which were found to inhibit anaerobic glucose metabolism are glucosone, 2-deoxyglucose, and glucosamine. These analogs differ from glucose in the substituted groups on the number 2 carbon.

Glucosone was found to be the most potent inhibitor of the anaerobic metabolism of all three glycolyzable sugars. It was especially potent

against fructose and was least effective against mannose. Glucosone was 23 times more potent that 2-deoxyglucose in suppressing glucolysis, the comparison being made on the basis of the relative concentrations required to give 50 per cent inhibition at a given glucose level.

2-Deoxyglucose also was most inhibitory against fructose and least inhibitory against mannose. It inhibited glucolysis 50 per cent when its concentration was equal to that of glucose. The inhibition appears to be of a competitive type since increasing the glucose concentration reversed 2-deoxyglucose inhibition.

The order of inhibition of fructose, glucose, and mannose by these inhibitors was inversely related to their relative affinities to hexokinase. Moreover, the respective degree of inhibition by glucosone, 2-deoxyglucose, and glucosamine corresponded approximately to their relative affinities to hexokinase.

In addition, eight other carbohydrate compounds were found to inhibit anaerobic fructolysis but to have no effect on glucolysis, even at the glucose-to-inhibitor ratio of 1 to 4. The inhibitors presented in decreasing order of potency are:

Mannoheptulose
3-*O*-Methylglucose
Glucoheptulose
1,5-Anhydroglucitol
Lyxose
Xylose
Galactose
1,5-Anhydromannitol

Each of these compounds, with the exception of 3-*O*-methylglucose, had been reported by Sols and Crane (9) to have a detectable affinity to brain hexokinase. The degree of inhibition, however, does not agree exactly with the relative affinity of each compound to the enzyme of brain tissue.

The compounds which were shown to have no inhibitory effect on tumor fructolysis and also no affinity to brain hexokinase are: ribose, sorbitol, *meso*-inositol, methyl-α-glucoside, and arabinose.

N-Acetylglucosamine had no effect on tumor fructolysis and glucolysis when present at 4 times greater concentration than either substrate. On the other hand, it was shown by Sols and Crane (9) to have an appreciable affinity to hexokinase and to be a strong competitive inhibitor for the phosphorylation of fructose. Presumably *N*-acetylglucosamine is unable to reach the site of hexokinase in the intact tumor cell. If hexokinase were localized on the ascites cell membrane, *N*-acetylglucosamine would be expected to be inhibitory.

STIMULATION OF GLYCOLYSIS BY CHEMICAL AGENTS

In the preliminary survey of the effects on metabolism of diverse chemical agents (6), it was noticed that some compounds are capable of stimulating anaerobic glucolysis. Although emphasis had been on finding new inhibitors, further investigation of stimulating agents was undertaken in an attempt to elucidate the factors which control the anaerobic metabolic rates in the intact cancer cell.

In his early observations, Warburg (2) found that the anaerobic glucolysis in slices of different malignant tumors varied from 13 to 37 microliters per hour on the conventional basis of milligram dry weight. Since ascites tumors consist of a nearly pure population of cancer cells, while slices do not, ascites tumors have a much higher glucolysis than slices, when calculated on a dry weight basis. In 1952, Warburg and Hiepler (10) reported that Ehrlich ascites tumors have an anaerobic rate varying from 45 to 55. The Krebs ascites tumor used in our work was found to have a similar range of rates (11).

In our investigations, anaerobic glucolysis of Krebs cells was stimulated to $Q_{CO_2}^{N_2}$ values as high as 117 without altering the intactness of the cells. This indicates that the high glucolytic rate of these cells is not a true maximum. That is, none of the eleven enzymes of the Embden-Meyerhof pathway is rate-limiting.

The following fifteen chemical agents were found to increase the anaerobic glucolysis of Krebs ascites cells:

2,4-Dinitrophenol
4,6-Dinitro-*o*-cresol
Gramicidin
Synthalin A
Triphenyl tetrazolium
Blue tetrazolium
Neotetrazolium
Acriflavine
Proflavine
Phenosafranine
Janus green B
Toluidine blue
Thionin
Podophyllotoxin
Tween 80

Dinitrophenol, which is one of the most potent of the stimulating agents, was studied under a variety of conditions. At a concentration of 3.0×10^{-4} *M*, dinitrophenol increased the glucolytic rate to the greatest extent. Concentrations both higher and lower than the above-mentioned level produced less stimulation. The degree of stimulation was independent of the concentration of glucose.

Fructolysis responded to dinitrophenol in a somewhat different manner than glucolysis. The effect of dinitrophenol on fructolysis was dependent upon the concentration of fructose. In the presence of fructose at a concentration of 30 millimolar or higher, fructolysis was stimulated to essentially the same extent as glucolysis. As the concentration of fructose was lowered, less increase was observed.

At the levels lower than 10 millimolar, no stimulation was obtained.

The metabolism of mannose, unlike that of glucose and fructose, was not stimulated by dinitrophenol at any level of mannose. In fact, mannose at high concentrations was found to suppress the stimulation of glucolysis by dinitrophenol. Mannose maintained its ability to inhibit glucose utilization, even in the presence of dinitrophenol.

Gramicidin, synthalin A, and triphenyl tetrazolium were found to resemble dinitrophenol in their effects on the metabolism of the different sugars, the major difference being the greater stimulation by dinitrophenol.

EFFECTS OF COMBINATIONS OF INHIBITOR AND STIMULATOR

Before combinations of inhibitor and stimulator were tried on glycolyzing cells, the following possible effects were anticipated: The rate would be (1) the same as with inhibitor alone, (2) the same as with stimulator alone, or (3) intermediate between the stimulated and inhibited rate. None of these possibilities proved to be correct.

When dinitrophenol was added together with glucosone, the inhibition of glucolysis was appreciably greater than with glucosone alone. This is a unique type of synergism.

2-Deoxyglucose in combination with dinitrophenol produced a somewhat greater inhibition than 2-deoxyglucose alone. However, when 2-deoxyglucose was added at a minimal inhibitory level the stimulation by dinitrophenol was only partly suppressed.

SUMMARY AND CONCLUSION

The effects of chemical agents on the rates of anaerobic metabolism of glucose, fructose, and mannose by Krebs ascites tumor cells were presented.

Glucosone and 2-deoxyglucose were found to be potent inhibitors of a competitive type of fructolysis, glucolysis, and mannolysis. The relative potency of these inhibitors was shown to be correlated with their affinities to hexokinase. Their relative effects on fructose, glucose, and mannose also appear to be related to the affinities of the glycolyzable substrates to hexokinase.

Among the 40 carbohydrate analogs tested, eleven were found to inhibit fructolysis; ten of these had been reported to have an affinity to hexokinase.

The results on stimulation indicate that none of the enzymes of the Embden-Meyerhof pathway is rate-limiting for either glucolysis or fructolysis. Phosphomannose isomerase, which is not involved in the metabolism of glucose and fructose, is suggested to be the rate-limiting enzyme for mannose metabolism.

Certain compounds were found to stimulate anaerobic glucolysis and fructolysis, of which dinitrophenol was shown to be one of the

most potent. Stimulation of glucolysis may be a result of activation of latent adenosinetriphosphatase in the cell. The degradation of ATP to ADP and inorganic phosphate and the formation of ATP may thereby be brought into optimal balance. This suggests that the level of inorganic phosphate or ADP limits the rate of unstimulated anaerobic glucolysis and fructolysis.

Stimulation appears to be dependent upon the presence of an excess of substrate. When the supply of substrate was blocked by a competitive inhibitor, such as glucosone, the rates of all the reactions of the glycolytic pathway were decreased, including the rate of formation of ATP. Under these conditions activation of ATPase may be expected to lower further the level of ATP and to produce a decrease instead of an increase in the rate of glucolysis.

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BOOK REVIEWS

DER KONTAKTUMFORMER, by Erich Rolf. 549 pages, illustrations, 7×10 in. Berlin, Springer Verlag, 1957. Price, DM 67.50 (approx. $16.00).

The author of this book is an authority in the field. His work in Germany and the United States is well remembered and his many friends in this country will never forget his thorough and systematic approach to any problem. His book is, in the best German tradition, a compendium, incorporating all the know-how covered as exhaustively as humanly possible. It is clear, simple and without superfluous words.

The subject of the book is the "Mechanical Rectifier with Commutating Reactors." Mechanical rectifiers for high power applications (i.e., high current and high voltage) are a favorite subject of unworkable inventions. Innumerable attempts were made, resulting always in arcs, explosions, or rapidly eroding devices. Floris Koppelman's "*Kontaktumformer*," however, was successful from the beginning because the physics of mechanical and electrical commutation were thoroughly investigated and understood before building the device. Led by the laws of nature, Koppelman used a series of startling and radical steps to achieve his goal, and was successful because he was fortunate enough to find the scientific and technical help he needed. A refreshingly new art evolved, coined by an individualistic and extremely competent inventor, and brought to ultimate completion by the author of this book. Main subject, therefore, is a complete description of a new art in its own right, involving various branches of engineering and physics which were heretofore not considered worthy of more careful investigation. Main contribution: Commutating reactors, i.e., reactors with magnetic cores of extreme nonlinearity, with a sound theoretical definition of what is required and how to obtain it.

Without explicitly mentioning it, this book also covers the general subject of the large, high efficiency, low impedance power converter. Devices of this kind have been made, using and embodying different physical principles. To propose such a device and build an experimental sample is already quite a task. But, to build many units in several countries with different industrial practices and to put them into operation and keep them operating successfully is an entirely different project. Unavoidably, rather novel effects are uncovered and new ways must be found to cope with them. High efficiency and low voltage drop of the converting means are very desirable, but hard to measure. It is less desirable to have a rigid, low impedance, tie-in between the a-c and the d-c system and an almost unsurmountable problem to prevent catastrophic fault currents in case of failure. The radical solution of applying a deliberate short circuit in the internal circuitry of the converter, within an extremely short time, and clearing it later by circuit breakers was a new solution. Very unorthodox methods were required and we are fortunate to find them well described.

Several chapters in the book are of special interest and should be consulted because they contain information which is not available in other publications. For example:

Electrical contacts operated in synchronism with the power frequency and switching very heavy currents while alternately withstanding high voltages, are a rather unknown circuit element. It is extremely valuable to understand them and, particularly, their behavior in closing and opening because of their incredibly high performance. These circuit elements are able to handle more than 1000 KVA of converter power per cubic inch of material and are extremely simple in design.

Confronted with a new means of power conversion, having its own peculiar properties, multiphase rectifier circuits, particularly the double way circuits, are thoroughly analyzed and tabulated.

Complex multiphase circuits, high power handling capacity, low impedance of the circuits and control problems require a

novel approach to the analysis of switching circuits. Here again, the approach is very original, but methodical and well explained. With the present interest in magnetic amplifiers, the special problems and solutions of the combination of synchronous contacts and rectangular hysteresis—loop, magnetic core devices are very welcome. Furthermore, the analysis of such switching circuits using the voltage-time integral approach and the clear separation of air core reactance (a geometric entity) from iron flux change (a property of materials) is a valuable and insufficiently publicized contribution in electrical engineering, developed by practical engineers and presented in a generally valid theory.

For those who are interested in the behavior of electric circuits, one of the most interesting parts of this book is the description of the commutating reactors and their control by means of pre-excitation and similar circuits. One of the major contributions to electrical engineering, by this art, is the deliberate change of shape in magnetizing currents and their compensation by electric circuits.

We are fortunate to have these new contributions available in a closed book form, well described both as a text and reference book. Much information of practical value is given, including complete examples of design, information on testing and operating power converters. The arrangement of the book, the printing and the illustrations are excellent. Literature, abbreviations and nomenclature are well tabulated.

EDWARD J. DIEBOLD
International Rectifier Corp.

ENGINEERING PROPERTIES AND APPLICATIONS OF PLASTICS, by Gilbert Ford Kinney. 278 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $6.75.

As an aid to those who need to apply the use of plastics to the fabrication of useful, marketable items, this book should be invaluable in fitting the proper material to the job.

During the formative stage of the plastics industry too many errors were made in choice of material, such as the thermoplastic birdcage spring that crept at room temperature during the night and left the birdcage on the floor in the morning.

This volume is written specifically for the designer and fabricator and will give a much better understanding of the materials and an ability to apply them properly, for, quoting from the book: "Proper use of plastics requires engineering design which, from its inception, recognizes the unique behavior of these materials."

After a short discussion of the general nature of plastics, their over-all advantages and disadvantages and present and future status as production materials, the book goes directly to the various types and forms, discussing each in detail, without going beyond the understanding of the designer and the fabricator.

For each type of material the discussion covers manufacture of the raw material, chemical and physical *reasons* for its characteristics and the variations in characteristics that can be produced through processing variations. Methods of fabrication, typical items produced from it and effectiveness of the material in its final, useful forms are discussed in much more detail than is generally found in publications intended for the final user. Thirteen different general types of plastics are discussed in sufficient detail to rate a chapter for each. Each of these, where necessary, is broken down into specific types whose characteristics are investigated.

Another chapter is devoted to molding methods. Here the advantages, disadvantages and proper application of the various methods of conversion into the final product are discussed. The effects of improper design and of improper control of the molding process are pointed out.

This chapter covers the methods of fabrication quite thoroughly, though the recently popularized "slush-casting" method is not discussed. It is barely mentioned back in one of the chapters on specific materials.

Mechanical, thermal, optical and electrical properties are given, each in a chapter devoted to it alone.

The chapter on chemistry of plastics is a little more sophisticated, to give details not generally found in simple discussions on the subject. It should be very valuable to the user in predicting behaviour of the

material during fabrication and during use of the final product.

Finally, a few tables of characteristics of the materials are given for ready reference in choosing a material for a specific purpose.

All in all, this is a book that should provide designers and molders with a tool that will help in avoiding some of the manufacturing errors committed in the past.

HERMAN P. SCHICK
The Franklin Institute Laboratories

FUNDAMENTALS OF ELECTRON DEVICES, by Karl R. Spangenberg. 505 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co. Inc., 1957. Price, $10.00.

Karl Spangenberg has become well known in the electrical engineering profession as one of Stanford University's great teachers. His earlier books are as popular as those of Terman and Skilling, also of the same institution.

The novelty of Professor Spangenberg's newest book resides in a sort of mixed composition by which the conventional vacuum tube and semi-conductor device are combined in the same chapters. That is, when diodes are discussed, the author takes up all types of diodes, vacuum and junction. When equivalent circuits are presented, those representing tubes are compared with those for transistors. Under small-signal amplifiers, vacuum tube and transistor circuits are analyzed together. In this way the resemblances as well as the dissimilarities are brought out.

This dual presentation of a former electron device with its contemporary solid state counterpart is certain to be a trend in textbooks now issuing. The student traversing this ground for the first time may thus be relieved of studying vacuum tubes and semiconductor devices successively and having to relate the two as best he can. This has been the problem of most electronics engineers originally trained in tube techniques.

Small-signal oscillators, non-linear effects, large-signal applications, photoelectric devices, and noise individually constitute chapters which combine their vacuum tube and semiconductor aspects. The first third of the volume is devoted to electric energy level concepts and electron effects before mention is made of the specific devices.

One minor criticism might be that Professor Spangenberg was apparently unwilling to stop with a treatment of the devices alone but felt some information on their applications was necessary. For his teaching requirements this may have been justifiable as a means of showing the subject's relation to other course material. However, from the standpoint of one using the book as a reference, the discussions of applications are too brief to be of value. Their inclusion tends to weaken the book's main theme, the fundamentals of electron devices. A vitally stronger presentation would have resulted through strict adherence to electron devices alone.

C. W. HARGENS
The Franklin Institute Laboratories

DESIGN OF STEEL STRUCTURES, by Edwin H. Gaylord, Jr. and Charles N. Gaylord. 540 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $8.00.

One of the better texts on structural design, this book is primarily concerned with the design of structural members as applied to bridges and building frames. Another in the McGraw-Hill series of books on civil engineering, it is written by E. H. Gaylord, Jr., Professor of Civil Engineering at the University of Illinois and C. N. Gaylord, Chairman of the Department of Civil Engineering at the University of Virginia. Basically, the authors thoroughly cover the design of all structural members comprising a bridge or building, punctuated with practical examples and even, in several cases, presenting the complete analysis of certain phases of existing bridges and buildings.

Logically, the authors introduce their subject with a discussion of the various loading conditions to which bridges and buildings are subject. Following this, types of bridges and buildings are described. Then follow descriptions of the detail parts of the structures: connections, tension members, compression members, beams, ending with the larger units, plate girders, industrial buildings and steel bridges. The treatment and coverage of the detail design are good; holes in webs and flanges, bending in connecting angles, pretensioning of anchor bolts, shear in lacing bars, lateral buckling of beams

seat connections, all are quite thoroughly explained. It's complete.

In recent years the theory of plastic design has become recognized as a valid method of designing structures. The authors have accordingly directed the last chapter to an introduction to the theory of plastic design.

E. W. HAMMER
The Budd Company

JETS, WAKES, AND CAVITIES, by G. Birkhoff and E. H. Zarantonello. 354 pages, illustrations, 6 × 9 in. New York, Academic Press Inc., 1957. Price, $10.00.

The book is intended to report the most important findings of nearly 100 years of research in the fascinating and complex field of jets, wakes and cavities. The emphasis of the monograph is devoted to the quantitative scientific analysis of these phenomena meaning that they are treated from the mathematical standpoint as a basis. Therefore the problems that have been successfully treated by this method are exhaustively described; whereas the more physical, descriptive research has been confined to a short chapter (15) at the end of the book.

The mathematical and physical foundations of the treatment of jets, wakes and cavities as well as the simple flow patterns of ideal liquids are extensively treated in Chapters 1 to 7. The change of flow pattern due to compressibility and gravity has been described in Chapter 8. Numerical computation is in Chapter 9; axially symmetrical flows are to be found in Chapter 10; unsteady potential flows are in Chapter 11; steady viscous wakes and jets in Chapter 12; periodic wakes in Chapter 13; and turbulent wakes and jets are found in Chapter 14. Mathematical results are completely summarized in the tables and curves at the end of the book, but the comparisons of experimental results with theoretical predictions are not as numerous as could be desired.

This book, which is the first systematic treatment of this field, can be a real help for researchers in the field; and as such, it is to be recommended. The field of jets has gained a considerable prominence in the ast few years, both in connection with jet propulsion and with explosive "shaped charges."

W. PHILIPPOFF
The Franklin Institute Laboratories

EXCITED STATES IN CHEMISTRY AND BIOLOGY, by C. Reid. 215 pages, diagrams, 5½ × 8½ in. New York, Academic Press, Inc., 1957. Price, $7.50.

Dr. Reid has performed a great service to the fields of biology and chemistry by presenting in a single treatise the manner in which quantum chemistry is applied to formerly little understood biochemical phenomena. The first three chapters deal with the fundamentals of reaction kinetics, electronic-energy levels and atomic and molecular spectra. The chapter on energy levels includes a review of the fundamentals of quantum theory and in a lucid manner makes plain the reasonableness of the quantum mechanical approach to the study of polyatomic molecules. The reader becomes acquainted in these first three chapters with the characteristics of the ground and excited states of molecules and is then in a good position to appreciate the discussions which follow in later chapters.

Excitation mechanisms including a brief summation of Kronig's selection rules for radiationless transitions are covered in chapter four.

Chapters five and six on the Triplet State and molecular energy exchange are among the most informative in the entire book. The author discusses the origin of phosphorescence in terms of a metastable triplet state and refers the reader to experiments in which actual triplet state life times have been estimated. Also there is included a discussion of the import and photo-oxidation reactions and it treats the "photodynamic action" effect which is involved in the lethal effects of "active oxygen" in certain biochemical systems.

Luminescent reactions (Chapter seven) which include, for instance, the mechanisms of the luminosity of bacteria and the firefly are presented in terms of the complicated luciferin-luciferase scheme. This is probably one of the most intriguing areas of non-medical biological research today. The eighth chapter deals with vision. The details

of the biochemical steps of vision are not completely known but the evidence is that unstable intermediates play an important role.

The effects of high energy radiations are treated in the ninth chapter and here the author explains the problems involved in research in this most important field. The complexity of the problem is immense due to the multiplicity of biological systems and a large spread of radiation dosages to be considered. Interesting and important work is mentioned of the effect of body temperature and added chemicals on the radiation resistance of organisms.

This book has references conveniently located at the end of each chapter and there are three appendices which give additional information on hamiltonian operators, group theory and time dependent Perturbation Theory.

The writer has succeeded in giving the reader an over-all insight into the wide range of "excited state" problems in biochemistry. This book is a welcome and important addition to the field of chemical physics.

JAMES B. DREW
The Franklin Institute Laboratories

LA DIFFUSION DANS LES MÉTAUX, edited by J. D. Fast, H. G. Van Bueren and J. Philibert. 124 pages, illustrations, 6 × 9 in. Te Eindhoven (Holland), Philips' Gloeilampenfabrieken, 1957. Price, $5.25.

A group of French metallurgists meet periodically to discuss current problems of metal physics. This book contains the eleven papers presented at the Eindhoven Conference, September 10 and 11, 1956, and all devoted to diffusion in metals. Two of the papers are concerned with the effect of structural defects (grain boundaries in particular) on metallic diffusion. Four papers are devoted to various aspects of the Kirkendall effect, and two to the diffusion of interstitial atoms. The last study considers the effect of elastic strain on the process of diffusion. Although relatively brief, these papers are clearly and comprehensively written and well illustrated. Most readers will appreciate the excellent introduction to the subject of diffusion, written by A. D. LeClaire. The literature on diffusion in the French language is relatively small, and this book constitutes an excellent addition to it. However, many of the English speaking metallurgists who understand French would find this small book profitable and instructive reading.

HENRI AMAR
The Franklin Institute Laboratories

TORNADOES IN ENGLAND: MAY 21, 1950, by H. H. Lamb. Geophysical Memoirs No. 99. 38 pages, illustrations, 8½ × 11 in. London, Her Majesty's Stationery Office, 1957. Price, 7s 6d.

This meteorological study is one of the series of geophysical memoirs printed by Her Majesty's Stationery Office in London. An extensive study of an extremely devastating series of tornadoes in the Buckinghamshire and Bedfordshire areas.

We of the United States are prone to feel that tornadoes are some sort of private disaster possessed only by us. Granted that the British storms described in the monograph only approach in severity the American type, such devastating whirlwinds are not to be treated lightly. The value of this work will be realized mostly by those meteorologically inclined who want to study a phenomenon of this sort in intensive fashion. It is extremely thorough and quite technical in most respects—a good case study.

R. W. NEATHERY
The Franklin Institute Museum

SYSTEM ENGINEERING, by Harry N. Goode and Robert E. Machol. 551 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $10.00.

The name systems engineering has meant different things to different people. To some it has implied a rather vague global view applied to complex combinations of equipment, the main virtue of which was that the practitioner was not required to know much about any detailed component of the combination. More serious practitioners of this engineering technique have attempted to define emergent characteristics of the system and have attempted to organize these characteristics in a quantitative manner.

It is in this sense that systems engineering has become a useful new technique, although it is questionable that it is a new engineering discipline.

Goode and Machol have done an excellent over-all job in providing the rudimentary techniques as well as productive insights into the analysis of large scale systems. Many examples of both a commercial and military nature are provided and, where appropriate, problems and their solutions are presented for the reader's instruction. The content of the book is divided approximately as follows: 40% to probability and statistics, 20% to computers, 20% to what is often referred to as operations research, 5% to servomechanism design and 5% to human factors and 10% to miscellaneous, though important matters. In this reviewer's mind the lack of emphasis on servomechanism theory is a large and glaring mistake in judgment. The human factors section, though of reasonable emphasis is inadequate in content. The few paragraphs on human operator control on pp. 496 and 497 are neither representative nor helpful.

Were the servomechanism control chapter expanded five fold, and the human engineering chapter rewritten the text would merit high praise.

EZRA S. KRENDEL
The Franklin Institute Laboratories

CORRECTIONS

On page 429 of the November 1957 JOURNAL, the listing for "An Introduction to Reactor Physics," by Littler and Raffle, erroneously stated that the U.S.A. edition was available from McGraw-Hill Book Co. Pergamon Press, the publisher, is distributing the book, at $5.50. The Editors were notified of the price and of the error in distributors too late to include the correct information with the review.

In the same issue, page 425, the publisher of "The Economics of Nuclear Power," was incorrectly listed. The correct publisher is Pergamon Press, of New York, London, Paris and Los Angeles.

CURRENT TOPICS

Aluminum Soldering Simplified.—Simple and effective techniques for soldering aluminum and its alloys as well as galvanized metals have been developed by G. M. Bouton and P. R. White, metallurgists at Bell Telephone Laboratories. These techniques employ an inexpensive and stable zinc base alloy as a preferred solder, and no flux or vigorous abrasion is necessary. Joints in aluminum made by these methods are stronger than commercial aluminum itself.

Because of their simplicity, the techniques are expected to find widespread applications both in industry and in the home workshop. They are most effective for those types of joints where the surfaces are accessible for manipulation with the solder stick, such as butt and "T" joints.

Long term stability of the soldered joint is assured by the rigid exclusion from the high purity zinc base alloy of deleterious elements such as lead, tin, bismuth and cadmium to prevent intergranular corrosion. A fraction of a per cent of magnesium may be added to enhance its stability, and up to several per cent of aluminum may be included.

In soldering aluminum it is not necessary to remove rolling mill oils ro the surface oxide from the area to be wetted. A single stroke of the solder stick across the heated aluminum surface will cause the solder to penetrate the oxide and wet the aluminum. The normally tenacious oxide film is lifted off much like paint peeling from wet wood. This raised oxide coating may then be wiped aside.

Surfaces thus wet can be joined by bringing them together and adding more solder by drawing the solder stick across the hot metal pieces. Heat may be applied electrically or by means of torches burning common fuels.

This soldering technique is equally effective for joining galvanized surfaces without a flux. Joints produced by this method are strong and stable.

List of Four-Letter Combinations Valuable for Coding Purposes.—The need for an efficient bibliographic sorting system for ready access to the thousands of scientific references collected in research laboratories has led to the development of a list of 10,000 randomly distributed four-letter combinations to be used as codes. The list was developed by Dr. William L. Porter, a chemist at the Eastern Utilization Research and Development Division, Philadelphia 18, Pa., a division of the USDA's Agricultural Research Service. These four-letter combinations have been used in the coding of bibliographic punch cards, but they have a wide potential application in all fields where large amounts of material must be coded for reference purposes. The list of four-letter combinations has been published with a detailed description of the generation of the code letters. Single copies of the publication, designated ARS-73-13, can be obtained from the division.

Thin-Screen X-Ray Amplifier for Medical Use.—A new thin-screen amplifier for X-ray viewing that multiplies by 100 times the brightness of images in certain types of medical X-ray examinations and may at the same time reduce exposure of the subject to X-ray radiation was demonstrated in experimental form to the American Roentgen Ray Society by a scientist of the Radio Corporation of America.

The device was described by Benjamin Kazan, of the technical staff at RCA's David Sarnoff Research Center at Princeton, N. J., as an electronic amplifying panel comparable in size and thickness to present conventional X-ray fluoroscope screens, but having these special capabilities:

It provides a stationary display of an X-ray image approximately 100 times brighter and with far greater visual contrast than those produced on present conventional screens, permitting immediate viewing in lighted surroundings rather than in the complete darkness and with lengthy dark-adaptation needed for viewing with the present equipment.

It holds its bright image for extended viewing up to 30 seconds after a short exposure to X-rays, and thus can cut down substantially the amount of X-ray exposure in the continued viewing of stationary images.

Incorporating a new electronic "erasing" technique, the experimental amplifying screen can be cleared at any moment in a fraction of a second and re-used immediately for viewing a new image, permitting rapid X-ray inspection of a subject from various angles, or the viewing of a series of "still" images of a moving subject.

In cases where the X-ray image is to be recorded by a camera, the combination of amplifying action and long persistence promises to be useful in allowing a reduction in X-ray exposure, since the film exposure can be continued after the X-rays have been cut off.

Mr. Kazan stated that at low X-ray levels several seconds are required for image build-up, but at higher levels the build-up is reduced to a fraction of a second, enabling the recording of a "temporary photograph" or "still" image of a moving subject. Although the build-up properties of the amplifying screen do not permit the continuous viewing of motion with low-level X-rays, the achievement of this end is the long-range objective of a continuing research program.

Medical Application Is Demonstrated

In Mr. Kazan's demonstration to the radiologists, the novel amplifier was operated without the usual room darkening. Shown on its 12-in. screen were bright X-ray images of a human femur, or thigh bone, containing a pin of the type used in hip-pinning operations, a simulated section of an intestine viewed from several positions, and a variety of small metallic objects. On a standard fluoroscope screen adjacent to the amplifier, a simultaneous display of similar objects was virtually invisible.

In an interview, Mr. Kazan pointed out that the characteristics of the experimental panel indicated its potential usefulness in a variety of applications.

"For example," he said, "a thin panel which presents a bright X-ray image after only a short exposure to the X-ray source and holds it up to half a minute, might be used to

provide immediate visual information for the surgeon during certain types of operation. This might be done in such operations as the pinning of a hip, in which the location of the pin must be carefully determined by observation. At the present time, such checking during the operation is normally accomplished by recording the X-ray image on film, hurriedly developing the film outside, and rushing it back into the operating room."

Discussing the technique of X-ray amplification, Mr. Kazan pointed out:

"This approach allows the retention of many of the desirable features of the conventional fluoroscope screen, since it permits the construction of the entire amplifying system in the form of a thin, large-area panel which can be easily viewed and manipulated, and which is free of complex auxiliary equipment."

How the Amplifying Screen Operates

He described the novel amplifying panel as a recent development in a continuing research program at RCA Laboratories. An earlier result of the program was an industrial amplifying fluoroscope demonstrated last year at the David Sarnoff Research Center. He gave the following account of the operating principles of the new amplifying panel:

It consists basically of a thin "sandwich" of two special materials in adjoining layers contained between transparent electrodes. One of these layers consists of photoconductive powder which conducts current only when it is exposed to X-ray or light radiation. The other layer consists of electroluminescent material, a type of phosphor which emits a bright light when an electric current is passed through it.

To operate the amplifier, a voltage is applied across the "sandwich" structure. When an X-ray pattern strikes the photoconductive layer, this material acts as an electrical "valve," allowing the current to pass through to the electroluminescent layer, which emits light corresponding to the X-ray pattern.

Besides these basic elements, according to Mr. Kazan, the experimental device includes a special "erasing" technique which cuts off the normal persistence of the screen whenever desired, leaving it immediately ready for the next X-ray pattern.

Ultrasonics Machines Brittle Ferrite.—Ultrasonic machining can solve problems encountered in shaping and machining of ferrite and other hard, brittle materials, according to the Sheffield Corporation, Dayton, Ohio, a subsidiary of Bendix Aviation Corporation.

Like glass, ceramics, germanium, silicon and other conductors, semiconductors or insulators, ferrite is extremely difficult to machine. This characteristic creates problems in shaping or cutting. The specific problem solved by The Sheffield Corporation's ultrasonic research division was to cut a slot 0.0015 in. wide through the wall of a ferrite toroid for insertion of a 0.001 in. thick silver pick-up. Such assemblies are used by thousands in electronic "brains." The problem of machining hundreds of such slots at a time is now under study.

Another ferrite application is the machining of an irregular shape into another type of ferrite toroid, to be used as a saturable reactor core. Previously machined on a jig bore using diamond wheels, the ferrite loaded the wheels, causing breakage and scrap losses. Ultrasonic machining provided the solution.

Here the problem also involved critical tolerances for a brittle material of this type, which was solved by Cavitron's ability to use tools in the softened state. This, combined with a new method of holding the toroids, made possible the shaping of multiple pieces at a time.

Ultrasonics again proved itself in the shaping of hard, brittle materials when Sheffield was asked to help in devising a production method for machining ferramic rings to required tolerances. A trepanning type tool was designed for the Cavitron so that slugs of the material would be removed, thus requiring a bare minimum of stock removal and giving maximum cutting rates.

Intricate and demanding problems in cutting and shaping difficult materials in a wide range of industrial processes are being met efficiently today by newly-developed applications of ultrasonic machining by Sheffield.

Washable Woolen Goods?—The lady of the house may soon be able to put all of her family's wool skirts, sweaters and trousers in the washing machine without fear of shrinkage due to felting. Working on a research project under the supervision of The Wool Bureau, Inc., the Harris Research Laboratories in Washington, D. C., have embarked on studies to learn how wool fabrics for all types of garments can be better "engineered" to resist felting shrinkage.

According to Dr. Gerald Laxer, head of the Bureau's Department of Science and Technology, properly "engineered" fabrics will probably require little or no special treatment to prevent felting shrinking when laundered in the latest types of domestic washing machines under mild conditions.

Studies have already shown that such factors as length and crimp of wool fibers, the thickness and twist of yarn, and especially the way a fabric is woven have marked influence on the felting shrinkage of wool fabrics when they are agitated during laundering. The aim of the investigation, which will run for two years, is to learn what combination of these fiber, yarn and fabric structure elements yield a variety of wool fabrics which will have a maximum resistance to felting shrinkage when they are washed in the newer types of washing machines.

Screw-Holding Screw Driver.—Hi-Fonic Music Systems, Inc. of Palo Alto, Calif., has found that the use of a new type of screw-holding screw driver has resulted in savings of more than 50 per cent in production time for assembly involving hard-to-reach places. The screw-holding screw driver, which is made by The H.J.J. Company, of Oakland, Calif., provides a quick, sure way to hold a screw in position. As the assembler pushes forward, tiny twin bits expand inside the slot of the screw. The screw will not be released until it is firmly seated in the thread.

Mr. Glenn Heaviside, production manager for Hi-Fonic, manufacturer of Ampex-licensed Tape Reproducers and complete sound systems for commercial application, reports that: "We have found H.J.J. screw-holders particularly handy for installing 400 Fillister Head machine screws in vertical positions, many of which are in fiendishly difficult spots to reach. They are a life-saver, and a disposition-saver, as well, for working along relays, and many other places where you cannot put the screw in place by hand. We also use the H.J.J. screw-holding driver extensively for

removing screws once they have been loosened by a standard screw driver."

The H.J.J. screw-holding driver is manufactured in a range of sizes and lengths to suit every requirement.

Lightweight Floating Bridge.—A hand-erectable floating bridge that can support 60-ton loads has been developed by the U. S. Army Engineer Research and Development Laboratories, Fort Belvoir, Va., and adopted by the U. S. Army and Marine Corps.

Commonly referred to as the M4T6, the bridge utilizes lightweight components that can be transported by air. A 750-lb. neoprene-coated nylon float is the heaviest single component.

The pneumatic half-floats join together to form a complete unit for use as a support at 15-ft. intervals. Deflated floats are stored and transported in canvas carrying bags.

Hollow aluminum alloy deck sections, less than 16 feet long and weighing 225 pounds each, are placed side by side in a staggered position to serve as a road surface. Steel beams and plywood panels are used to provide stiffness and distribute the load to the floats.

The bridge can be manually erected at rates up to 1½ ft. per minute. Construction can be speeded by the use of newly-developed bridge erection aids, such as a tilting bed trailer that carries and launches a complete bay with ease.

The bridge was subjected to exhaustive tests at Prince, West Virginia, and in Europe before it was adopted. Project engineer Seymour Wengrovitz, a graduate of the City College of New York, served as a technical advisor during the troop tests in Europe.

Industrial Bright Beam Lamp.—A new lamp gives brilliant and uniform illumination, free from shadows and bright spots. The Industrial Bright Beam Lamp, a new kind of industrial lamp, optically designed for all close-work operations, is said to increase production and reduce errors and spoilage. Ruggedly constructed, vibration resistant, this lamp features a mechanical adjustability that enables it to be placed in practically any position for convenient and best illumination. It is useful for layout, profile grinding and milling, engraving, filing, sharpening, assembly of small parts, etc. and for inspection of surfaces, finishes, threads, optical surfaces, textiles. It can be used in conjunction with stereoscopic and dissecting microscopes, toolmakers' microscopes, magnifiers and loupes and with student microscopes. It is also a fine source of illumination for opaque materials such as minerals and metallurgical specimens under ordinary microscopes. It throws a variable spot of light from ⅝ in. in diameter, very intense, to about 3 in., less bright, simply by sliding focus tube back and forth. The lamp is available on direct order from Edmund Scientific Co., Barrington, N. J.

Snow Pit Yields History of Antarctic.—A deep snow pit dug for scientific purposes at the Amundsen-Scott IGY South Pole Station reached a depth of 50 ft. on October 1, according to reports reaching the IGY Committee of the National Academy of Sciences.

The pit, which was dug by hand throughout the long Antarctic winter, serves a dual purpose. The successive levels of snow laid bare form an unspoiled record of climatic and other history. And the snow itself is melted for the Station's water supply.

Snow in the pits is so hard that even saws cannot be used. The snow must be chipped loose with mattocks or ice axes and then shoveled into bags and hauled to the surface. Each man at the Station spends a minimum of two hours every week either cutting or hauling snow, despite the rapid onset of fatigue in the sub-zero temperature. An 18° ramp is maintained to provide access to the pit.

Reports from the Amundsen-Scott Station state that the temperature in the pit is nearly constant at −60° F., while at the surface a record temperature reading of −102.1° F. was recorded on September 17. During the period from May 11 to September 17 South Pole temperatures were lower than −95° F. seventeen times.

A careful study of snow stratification, combined with examination of snow crystals and density, yields a history of the Antarctic. Minute traces of ash may indicate an earthquake hundreds of years ago. Pollen deposits provide a clue to past wind system.

Measurements are being made to determine whether the Antarctic ice sheet is experiencing an annual gain or loss of thermal energy. This work involves measurements of the temperature profile, heat capacity, aud thermal conductivity of the ice with depth.

IGY Scientific leader at the Pole Station is Dr. Paul Siple. Eight other scientists and a similar number of Naval personnel man the Station. Logistic support for the IGY activity is provided by Task Force 43, US Navy, commanded by Rear Admiral George Dufek.

Satellite Telescope Being Used by Moonwatch Groups.—The Moonwatch stations around the U.S.A. are equipped with special wide field telescopes. When the stations were originally planned, a search was made for an existing low power, wide field telescope at low cost. No existing suitable instrument was found.

The search led to the Edmund Scientific Co., Barrington, New Jersey, from whose large stock of war surplus telescopes and optics a six element war surplus wide field Erfle eyepiece was selected. This was used in conjunction with a wide diameter gunsight objective to make a 5½ power, 12 degree field telescope with a large exit pupil for night use. With it, stars fainter than the ninth magnitude can be seen.

The Moonwatch Satellite Telescope must be specially mounted for the requirements of group viewing. In front of the objective a first surface mirror is placed to provide easy viewing of the heavens. Also, each scope must hinge at the mirror so that when group observing is done no area in the sky is missed.

As the Satellite travels by at 18,000 miles per hour, it is only in view for a number of seconds. Also, since its exact path of travel is not known prior to passage, a telescope with a wide field is essential. A pair of 6 × 30 binoculars has a field of about 8½° and 7 × 50 binoculars about 7°. The inexpensive field glasses sold for a few dollars and opera glasses have fields of only 2° to 4°.

Therefore, field and opera glasses are of little or no use. The 6 × 30 binoculars have an exit pupil of 5 mms. and the 7 × 50 an exit pupil of 7 mm. As the Satellite is visible only at dawn and dusk, the 7 × 50 are, therefore, preferred because of their greater light gathering power even though their field of view is smaller.

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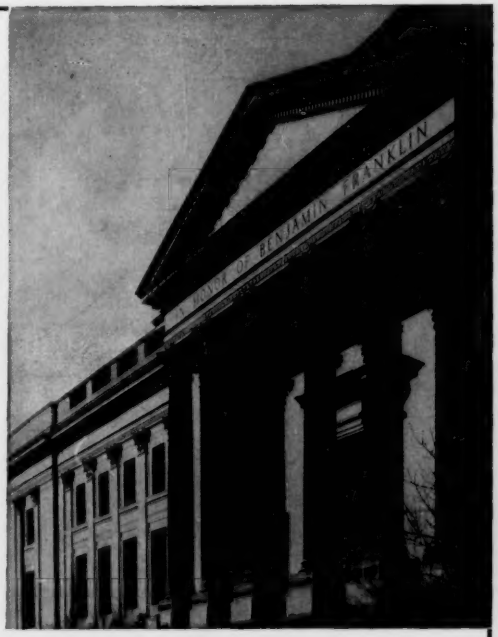

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